Math Review

Friday, January 3, 2025 6:13 AM

1. Solve the system of equations for A:

$$7A - 5B = -1$$
$$B + A = 5$$

$$5B + 5A = 25$$

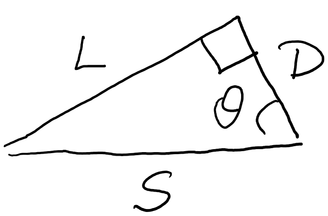
$$7A - 5B + 5B + 5A = -1 + 25$$
$$12A = 24$$
$$A = 2$$

2. Solve the equation $x^2 + 4x = 96$.

$$ax^2 + bx + c = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

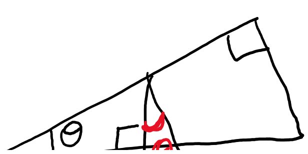
$$x^2 + 4x - 96 = 0$$
$$x = \frac{-4 \pm \sqrt{4^2 - 4(-96)}}{2}$$
$$x = \frac{-4 \pm \sqrt{16 + 384}}{2} = \frac{-4 \pm \sqrt{400}}{2} = \frac{-4 \pm 20}{2}$$
$$x_1 = -12 \qquad x_2 = 8$$

3.

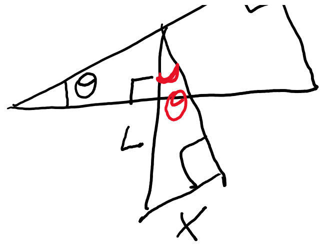


$$\frac{D}{S} = \cos\theta \qquad D = S\cos\theta$$
$$\frac{L}{S} = \sin\theta \Rightarrow \qquad L = S\sin\theta$$

4.



$$\frac{X}{L} = \sin\theta$$
$$L\sin\theta$$



$x = L\sin\theta$

5. $f(x) = 2x^{-3}$ $\quad \frac{df}{dx} = 2\cdot(-3)x^{-4} = -6x^{-4}$

$f(x) = 2x\sin(5x^2)$

$f'(x) = 2\sin(5x^2) + 2x\cdot 10x\cos(5x^2)$

$= 2\sin(5x^2) + 20x^2\cos(5x^2)$

6. $f(x) = \frac{1}{x^2}$ $\quad \int \frac{1}{x^2}\,dx = -\frac{1}{x} + C$

$f(x) = x^7$ $\quad \int x^7\,dx = \frac{x^8}{8} + C$

1. A particle is moving along the *x*-axis. Its position as a function of time is given as

$x = bt - ct^2$.

a) What must be the units of the constants *b* and *c*, if *x* is in meters and *t* in seconds?
b) At time zero the particle is at the origin. At what later time *t* does it pass the origin again?
c) Derive an expression for the *x*- component of velocity.
d) At what time *t* is the particle momentarily at rest?
e) Derive an expression for the *x*-component of the particle's acceleration, a_x.

a) $[b] = \frac{m}{s}$ $[c] = \frac{m}{s^2}$

b) $x = bt - ct^2 = 0$

$t_1 = 0$ t_2: $b - ct_2 = 0$

$t_2 = b/c$

c) $v_x = \frac{dx}{dt} = b - 2ct$

d) $v_x = b - 2ct = 0$

$t = \frac{b}{2c}$

e) $a_x = -2c$

2.

Friday, January 3, 2025 7:36 AM

A particle is moving along the *x*-axis. Its acceleration is given by $a_x = ct - dt^2$

At *t*=0, the particle is at rest at the origin.

a) Derive equations for position and velocity as functions of time.

b) What is the maximum velocity the particle reaches?

$$a_x = \frac{dv_x}{dt}$$

$$v_x = \int a_x \, dt = \int (ct - dt^2)\, dt = \frac{1}{2} ct^2 - \frac{1}{3} dt^3 + K$$

$$@\ t=0: \ v_x = 0 \Rightarrow K = 0 \qquad \boxed{v_x = \frac{1}{2} ct^2 - \frac{1}{3} dt^3}$$

$$x = \int v_x \, dt = \int \left(\frac{1}{2} ct^2 - \frac{1}{3} dt^3\right) dt = \frac{1}{6} ct^3 - \frac{1}{12} dt^4 + K_2$$

$$@\ t=0: \ x = 0 \Rightarrow K_2 = 0 \qquad \boxed{x(t) = \frac{1}{6} ct^3 - \frac{1}{12} dt^4}$$

$$v_x \, max: \ \frac{dv_x}{dt} = 0 \Rightarrow a_x = 0$$

$$ct - dt^2 = 0$$

$$t = 0 \quad or \quad t = \frac{c}{d}$$

origin, stopped

Two horses

Wednesday, July 29, 2015 7:40 AM

3. The figure shows position vs time graphs for two horses.

a) Sketch velocity vs time graphs for each of the horses.

b) Do the horses ever have the same speed? Where?

c) Does horse A ever passes horse B? If so, indicate at which point in time.

d) Does horse B ever passes horse A? If so, indicate at which point in time.