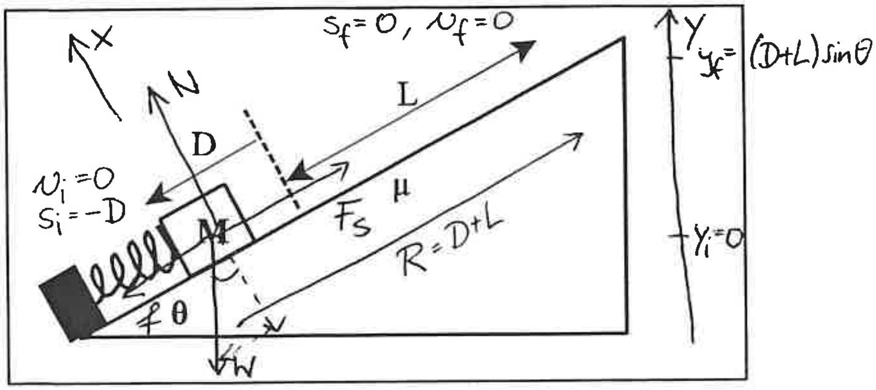
7) A box of mass M is on a rough incline that makes an angle θ with the horizontal. The coefficient of kinetic friction between the box and the incline is μ. The box is placed against a spring whose other end is secured to a wall at the lower end of the incline. The block is used to compress the spring a distance D and is then released from rest.



a)(10) Complete the diagram with all information necessary to solve Part b) below. Remember, any algebraic quantity that you use must appear in the diagram. You may want to add elements as you go along.

(OSE) b)(40) In terms of relevant system parameters, derive an expression for the minimum spring constant k necessary to ensure that the box reaches a distance L up the incline **from the equilibrium position of the spring**. (Treat the box as a point mass.)

$$E_f - E_i = W_{other}$$

$$\frac{M}{2}\cancelto{0}{v_f^2} + Mg y_f + \frac{1}{2}k\cancelto{0}{s_f^2} - \left(\frac{M}{2}\cancelto{0}{v_i^2} + Mg\cancelto{0}{y_i} + \frac{1}{2}k s_i^2\right) = \cancelto{0}{W_N} + W_{fr}$$

$$Mg(D+L)\sin\theta - \frac{1}{2}k(-D)^2 = \vec{f}\cdot\vec{R} = -(\mu N)(D+L)$$

to find N: $\sum F_x = N_x + W_x + \cancelto{0}{f_x} + \cancelto{0}{F_{Sx}} = M\cancelto{0}{a_x}$

$$N + (-Mg\cos\theta) = 0$$

$$N = Mg\cos\theta$$

$$Mg(D+L)\sin\theta - \frac{1}{2}kD^2 = -\mu Mg\cos\theta\,(D+L)$$

$$\frac{1}{2}kD^2 = Mg(D+L)\sin\theta + \mu Mg(D+L)\cos\theta$$

$$\boxed{k = \frac{2Mg(D+L)[\sin\theta + \mu\cos\theta]}{D^2}}$$

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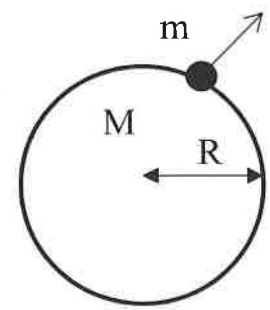
Name: Solution

7. a) OSE (30 points) A projectile of mass m is shot directly away from the surface of a planet of mass M and radius R with an initial speed that equals **½ the escape speed** from the planet. Derive an expression for the maximum distance from the center of the planet the projectile reaches, in terms of R.



1. Find escape speed:

$$E_f - E_i = \cancel{W_{other}}^{\,0}$$

$$E_i = E_f$$

$$\frac{1}{2} m v_{esc}^2 - \frac{GMm}{R} = \frac{1}{2} m \cancel{v_f}^{2\,0} - \cancel{\frac{GMm}{r_f}}^{\,0} = 0$$

$$\boxed{v_{esc} = \sqrt{\frac{2GM}{R}}}$$

2. Launch with $\frac{1}{2}\, v_{esc}$:

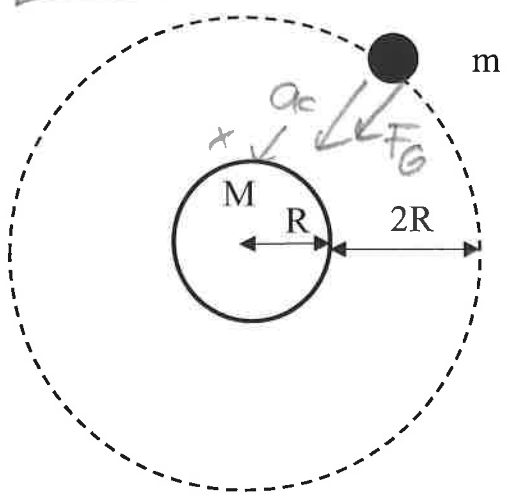
$$E_f - E_i = \cancel{W_{other}}^{\,0}$$

$$E_i = E_f$$

$$\frac{1}{2} m \left(\frac{1}{2}\sqrt{\frac{2GM}{R}}\right)^2 - \frac{GMm}{R} = \frac{1}{2} m \cancel{v_f}^{2\,0} - \frac{GMm}{x}$$

$$\frac{1}{4}\frac{GMm}{R} - \frac{GMm}{R} = -\frac{GMm}{x} \longrightarrow \boxed{x = \frac{4}{3}R}$$

b) OSE (20 points) A satellite of mass m is put into a circular orbit around the planet of mass M and radius R. It orbits a distance $2R$ above the planet's surface. Derive an expression for total mechanical energy E of the satellite, in terms of G, m, M and R.



$$E = K + U = \frac{1}{2} m v^2 - \frac{GMm}{3R}$$

Find v: $\Sigma F_x = F_{Gx} = m a_x$

$$\frac{GMm}{(3R)^2} = m\frac{v^2}{3R}$$

$$v^2 = \frac{GM}{3R}$$

$$E = \frac{1}{2} m \frac{GM}{3R} - \frac{GMm}{3R}$$

$$\boxed{E = -\frac{GMm}{6R}}$$

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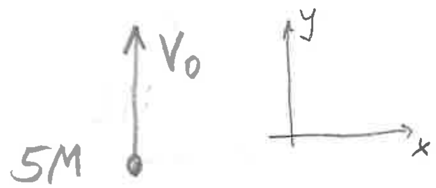
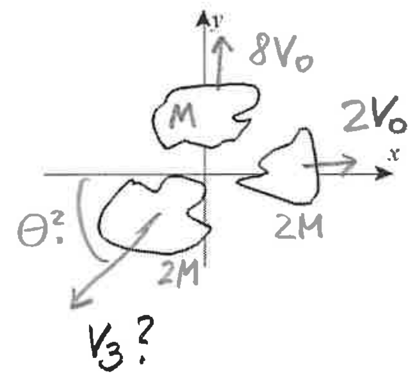
8. A star cruiser of mass 5M and travelling in the positive y-direction at speed V_0 is in transit half way between The Milky Way and Galaxy NGC 300. Due to an existential flaw in the dilithium crystals used as fuel, the star cruiser explodes into three fragments. One fragment of mass 2M moves in the positive x-direction with speed $2V_0$. The second fragment of mass M moves in the positive y-direction with speed $8V_0$. The third fragment moves in an unknown direction.

a) (10 points) In the diagram add all information to solve part b below.

Before



After



b)(OSE) (30 points) Derive an expression for the velocity of the third fragment, in terms of system parameters. Express the velocity in **unit vector notation**.

$$\cancel{\vec{J}_{net\ ext}} = \vec{P}_f - \vec{P}_i$$

$$P_{ix} = P_{fx} \qquad P_{iy} = P_{fy}$$

$$0 = 2M \cdot 2V_0 + 2M V_{3x} \qquad 5M \cdot V_0 = M \cdot 8V_0 + 2M V_{3y}$$

$$V_{3x} = -2V_0 \qquad V_{3y} = -\frac{3}{2} V_0$$

$$\boxed{\vec{V}_3 = -2V_0\,\hat{i} - \frac{3}{2} V_0\,\hat{j}}$$

c) (10 points) Determine the angle the velocity of the third fragment makes with the negative x-axis. Simplify as far as possible.

Part c not assigned

$$\theta = \arctan \frac{|V_{3y}|}{|V_{3x}|} = \arctan \frac{\frac{3}{2}V_0}{2V_0}$$

$$\boxed{\theta = \arctan \frac{3}{4}} \quad \text{or } 36.9^\circ$$

(if you remember the angles in a 3-4-5 triangle)

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