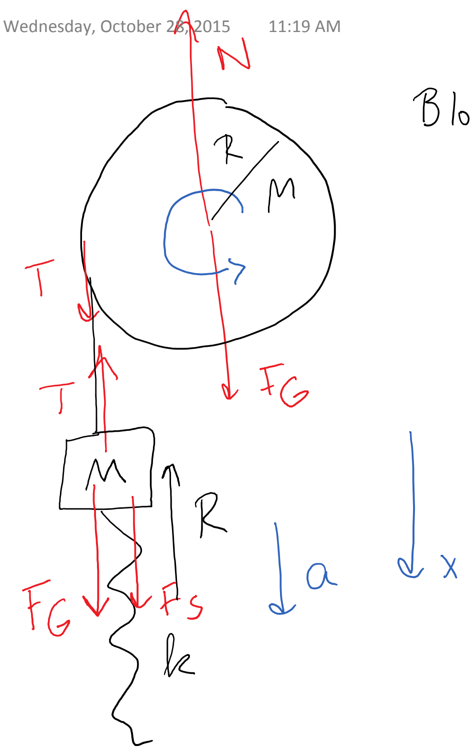
1a

Wednesday, October 28, 2015 11:19 AM



Block: $\sum F_x = T_x + F_{Gx} + F_{Sx} = Ma_x$

$$-T + Mg + kR = Ma_x$$

Note: $F_{Sx} = -kx$ on eq. sheet refers to x as $\ell - \ell_{eq}$, stretch of spring, NOT component along an axis. Use $|F_S| = kR$, decide sign based on axis.

Pulley: $\sum \tau_z = \cancel{\tau_{N_z}}^{0} + \cancel{\tau_{G_z}}^{0} + \tau_{T_z} = I\alpha_z$

$$TR = \frac{1}{2} MR^2 \alpha_z$$

No slip: $a = \alpha R$

$$T = \frac{1}{2} Ma$$

$$Mg + kR = Ma + \frac{1}{2} Ma$$

$$\boxed{a = \frac{2}{3}\,\frac{Mg + kR}{M}}$$

Note: z-axis and x-axis should count same motion as positive. If $\circlearrowleft_z$ and $\uparrow x$,

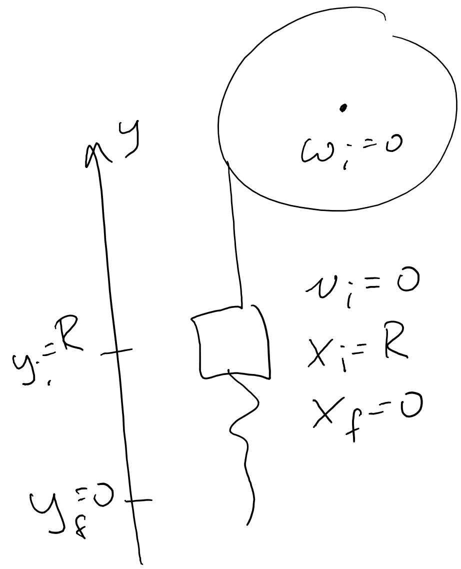
$$a_x = -\alpha_z R$$

Students will miss this minus sign.

So, make $\circlearrowleft_z$ $\downarrow x$

1b

Wednesday, October 28, 2015 11:26 AM



$$E_f - E_i = \cancel{W_{other}}^{\,0}$$

No slip:
$v = \omega R$

$$E_i = E_f$$

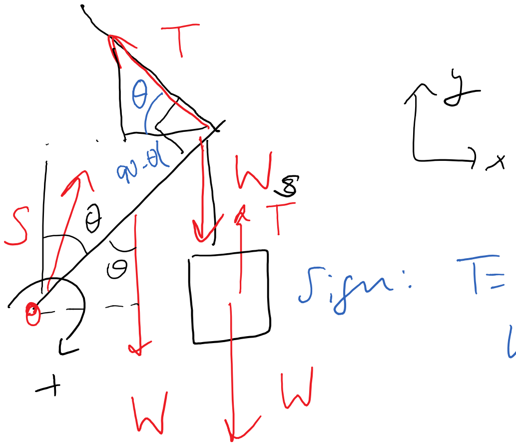
$$\tfrac{1}{2} M \cancel{v_i^2} + \tfrac{1}{2} I \cancel{\omega_i^2} + M g y_i + \tfrac{1}{2} k x_i^2 =$$

$$\tfrac{1}{2} M v_f^2 + \tfrac{1}{2} I \omega_f^2 + M g \cancel{y_f} + \tfrac{1}{2} k \cancel{x_f^2}$$

$$M g R + \tfrac{1}{2} k R^2 = \tfrac{1}{2} M v_f^2 + \tfrac{1}{2}\left(\tfrac{1}{2} M R^2\right)\left(\frac{v_f}{R}\right)^2$$

$$= M \underbrace{\left(\tfrac{1}{2} + \tfrac{1}{4}\right)}_{3/4} v_f^2$$

$$\boxed{v_f = \sqrt{\frac{4}{3}\,\frac{M g R + \tfrac{1}{2} k R^2}{M}}}$$



Sign: T = W ⇒ a force of magnitude W_{sign} acts on beam

$$\sum \tau_z = \cancel{\tau_{S_z}}^{\,0} + \tau_{W_z} + \tau_{W_{S_z}} + \tau_{T_z} = 0$$

nor

$$+ W \frac{L}{2} \sin\theta + W L \sin\theta - T L \underbrace{\sin 90^\circ}_{1} = 0$$

$$\boxed{\frac{3}{2} W \sin\theta = T}$$

$$\sum F_x = S_x + \cancel{W_x} + \cancel{W_{S_x}} + T_x = 0$$

$$S_x - T\cos\theta = 0$$

$$S_x = T\cos\theta = \frac{3}{2} W \sin\theta\cos\theta$$

$$\sum F_y = S_y + W_y + W_{S_y} + T_y = 0$$

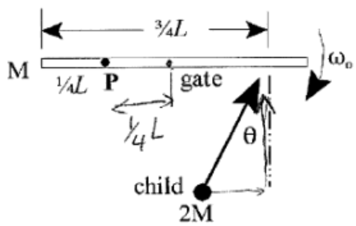
$$S_y - 2W + T\sin\theta = 0$$

$$S_y = 2W - T\sin\theta = W\left(2 - \frac{3}{2}\sin^2\theta\right)$$

Petting zoo

Wednesday, October 28, 2015 12:11 PM

9.(30) A entrance gate into a petting zoo has mass M and width L. It is pivoted at point **P** that is $\frac{1}{4}L$ from one end. A parent has just gone through the gate and it is swinging back. A child of mass $2M$ enters the zoo by leaping onto the gate with speed V and angle θ at a point that is $\frac{3}{4}L$ from its end. **Just before** the child hits and clings to it, the gate's angular speed is ω_o as shown in the diagram. The moment of inertia of the gate about its center of mass is $^1/12\, ML^2$ and about an end it is $\frac{1}{3}ML^2$.



a)(10) What is the moment of inertia of the gate about its pivot **P**?

$$I_P = I_{CM} + Mh^2 = \frac{1}{12}ML^2 + M\left(\frac{1}{4}L\right)^2 = \left(\frac{1}{12}+\frac{1}{16}\right)ML^2$$

$$= \left(\frac{4}{48}+\frac{3}{48}\right)ML^2 \qquad \boxed{I_P = \frac{7}{48}ML^2}$$

b) (20) Derive an expression of the angular speed of the gate-child system just after the child has leaped onto it.

$$\tau_{net_z} = 0 \rightarrow L_{iz} = L_{fz}$$

$$I_i\,\omega_{oz} - 2MV\tfrac{1}{2}L\cos\theta = I_f\,\omega_{fz}$$

$$\frac{7}{48}ML^2\omega_o - 2MV\tfrac{1}{2}L\cos\theta = \underbrace{\left(\frac{7}{48}ML^2 + 2M\left(\tfrac{1}{2}L\right)^2\right)}_{\frac{31}{48}ML^2}\omega_{f_z}$$

$$\boxed{\omega_f = \left|\frac{\frac{7}{48}ML^2\omega_o - MVL\cos\theta}{\frac{31}{48}ML^2}\right| = \left|\frac{7L\omega_o - 48V\cos\theta}{31L}\right|}$$