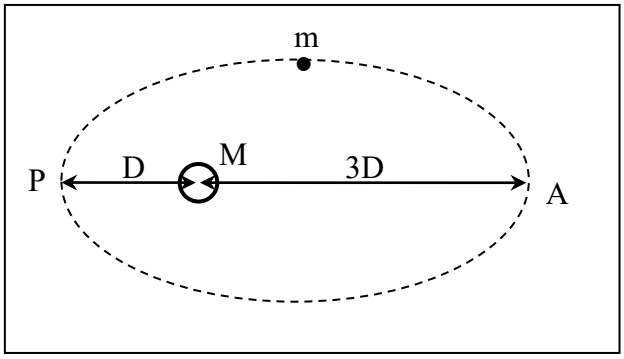
Physics 1135 Homework #16: Gravitational potential energy

1. A comet of mass *m* is in an elliptical orbit around a star of mass *M*. At the closest point of its orbit, point P, the comet is a distance *D* from the center of the star, and at the farthest point, point A, the distance is 3*D*. You may disregard the presence of all other celestial bodies.



Derive an expression for the change in the comet's **kinetic energy** as it moves from point P to point A, in terms of system parameters and constants.

Does the comet have its greatest speed at point A or point P?

2. A planet has mass *M* and radius 2*R*.

a) Derive an expression for the escape speed from the planet.

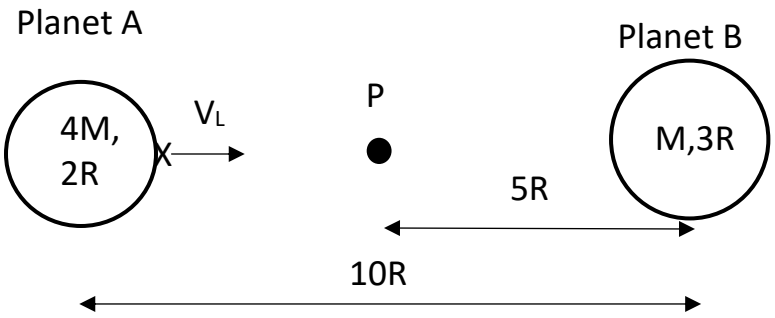
b) A projectile of mass *m* is shot directly away from the surface of the planet **at ⅓ of the escape speed** from the planet. Derive an expression for the maximum distance from the center of the planet the projectile reaches, in terms of *R*. Simplify as far as possible. (Ignore the existence of all other celestial objects.)

3. A spaceship of mass *m* has its engines switched off and is **moving in a circular orbit** at height *R* above the surface of a planet of mass *M* and radius *R*.

a) Derive an expression for total mechanical energy *E* of the orbiting spaceship, in terms of *G*, *m*, *M* and *R*.

b) Derive an expression for the minimum speed *V* the spaceship would need to escape from this orbit into deep space, in terms of system parameters. (The engines can't fire for the whole trip; they can only give the spaceship one boost so it obtains this velocity. Ignore all other celestial objects.)

4. The kings of planet A (mass $4M$, radius $2R$) and planet B (mass M, radius $3R$) want to meet for negotiations. The planets are a distance $10R$ from one another, center to center. For absolute fairness, the kings (who possess no physics knowledge) decide that the meeting place **P** is to be exactly halfway between the planets. A space capsule of mass m is launched from point **X** on the surface of planet A by means of a giant cannon, which gives it a launch speed V_L. It travels directly along the line that connects the centers of both planets. Ignore the orbital motion of the planets.



a) Derive an expression for the speed V with which the capsule arrives at the meeting place **P**, in terms of relevant system parameters.

b) Derive an expression for the net force (magnitude and direction) experienced by the capsule when it is at point **P**.

c) At what distance from planet A is the net gravitational force zero?