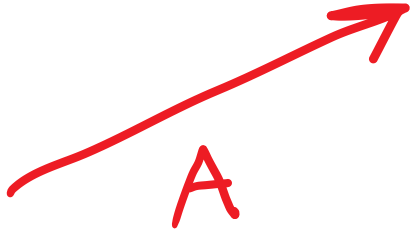
Lecture 3: Vectors and 2-d Kinematics

- Definition
- Unit vector notation, components, magnitude and direction
- Addition and subtraction of vectors in unit vector notation
- Position, velocity, and acceleration in 2-d
- Separation of motion in x-and y-direction

Vectors

A vector is a quantity that has size (magnitude) and direction. It can be symbolized by an arrow.



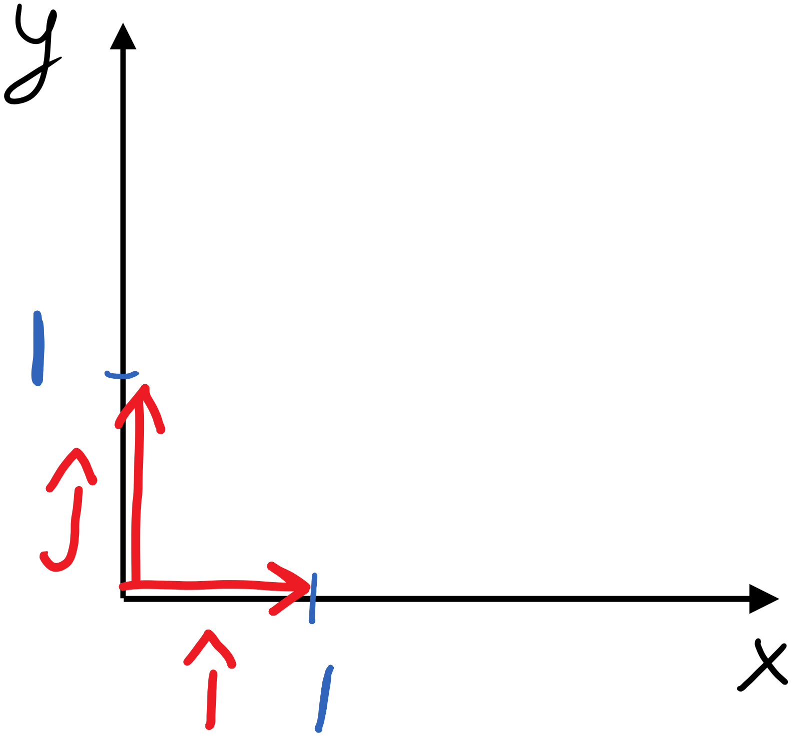
Length of the arrow represents magnitude

Notation convention:

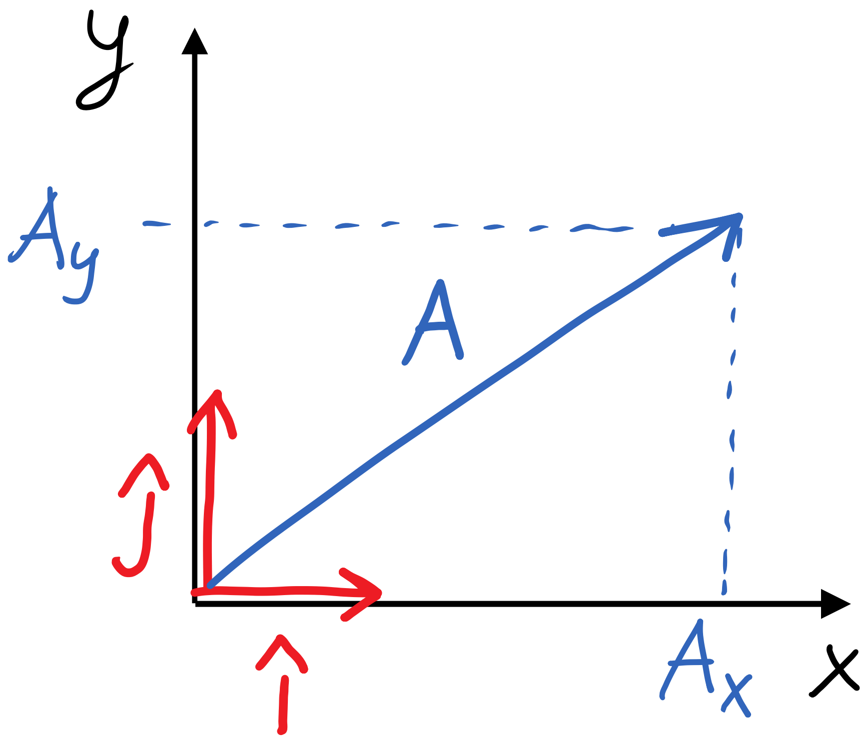
$\vec{A}$ denotes vector of magnitude $A = |\vec{A}|$

*Sometimes bold-face type also indicates a vector – hard to do in handwriting

Unit vectors

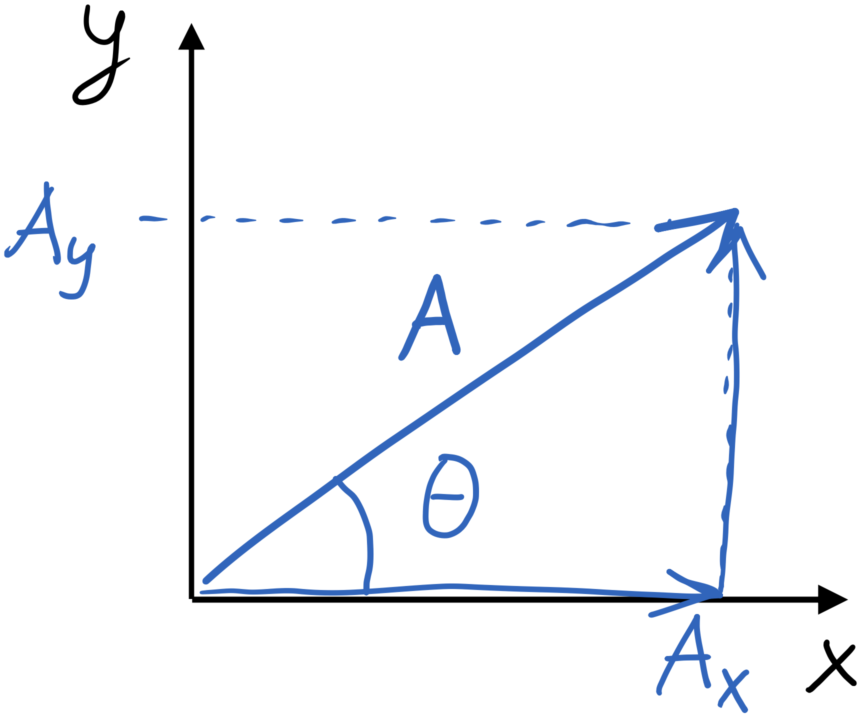


Unit vector notation



$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$

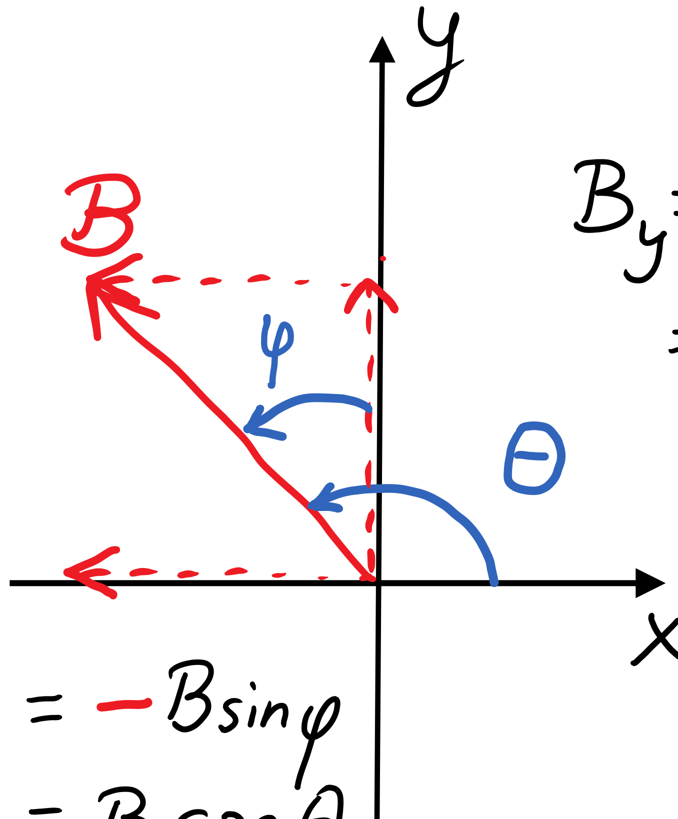
Vector components



$$A_x = +A cos\theta$$
$$A_y = +A sin\theta$$

$\vec{A} = A\cos\theta\,\hat{i} + A\sin\theta\,\hat{j}$

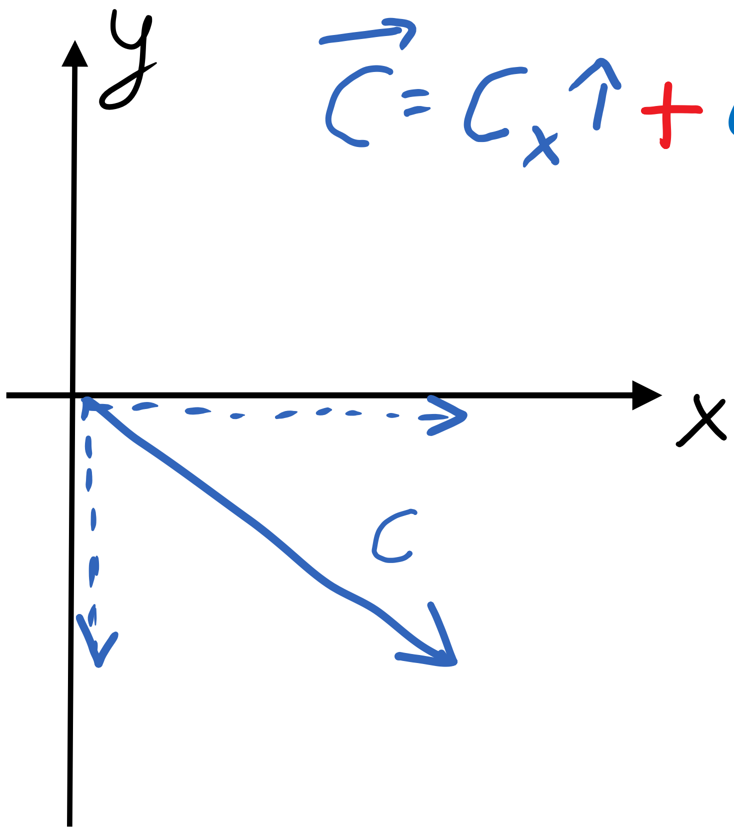
Vector components



$$B_y = +B\cos\varphi = B\sin\theta$$

$$B_x = -B\sin\varphi = B\cos\theta$$

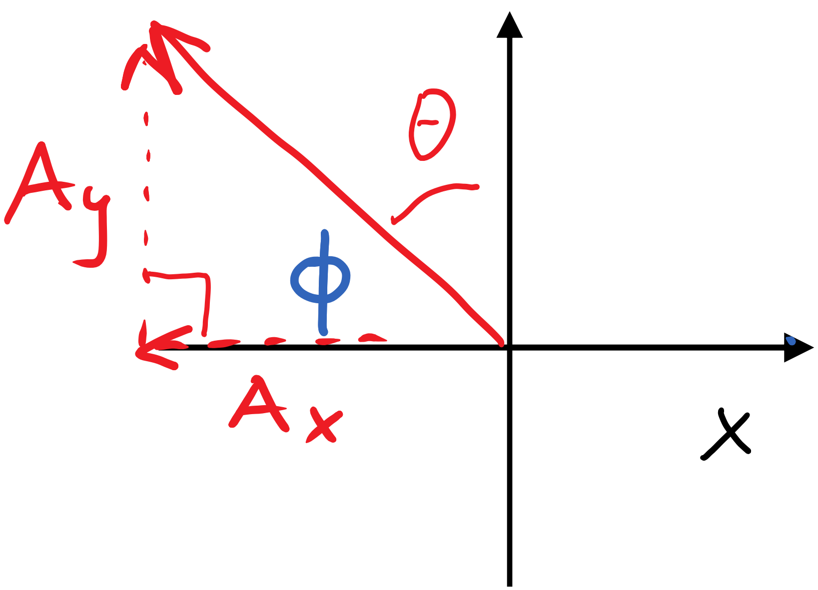
Unit vector notation



$$\vec{C} = C_x \hat{i} + C_y \hat{j}$$

C_y is negative

Magnitude and direction



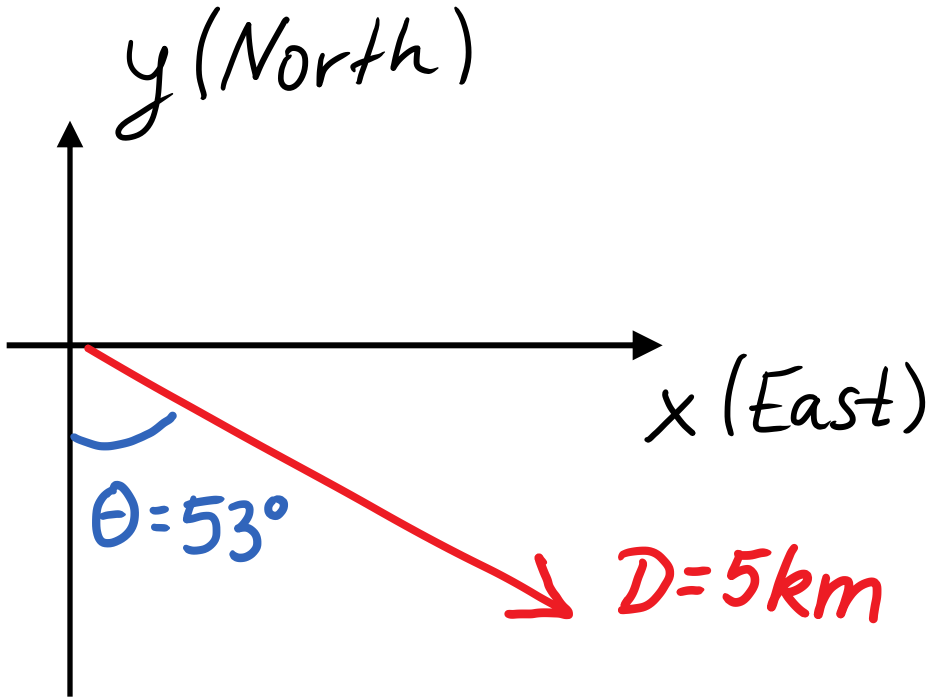
$$A = \sqrt{A_x^2 + A_y^2}$$

$$\tan\theta = \frac{|A_x|}{|A_y|}$$

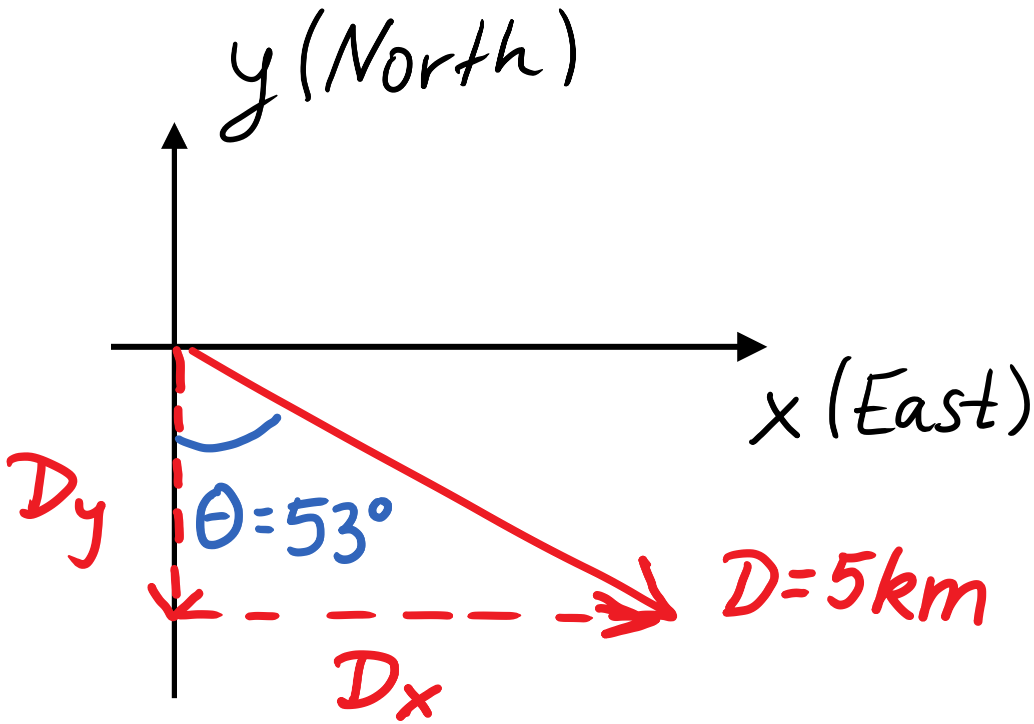
$$\tan\phi = \left|\frac{A_y}{A_x}\right|$$

Example

A displacement of 5 km is directed θ = 53° East of South. What is the displacement vector in unit-vector notation?

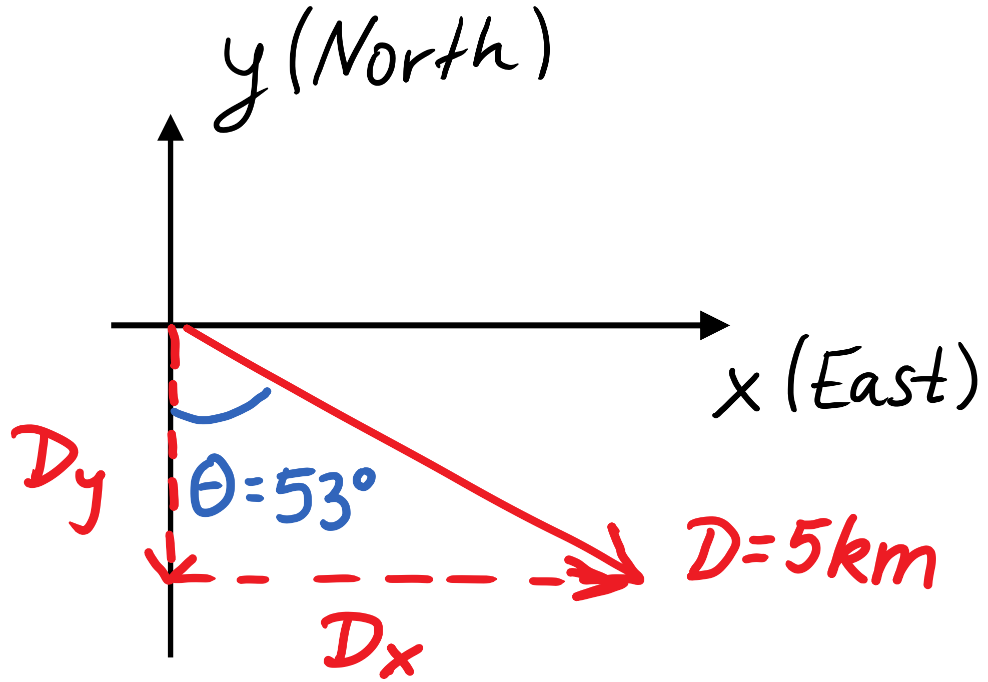


Example



$$\vec{D} = D_x \hat{i} + D_y \hat{j}$$

Example

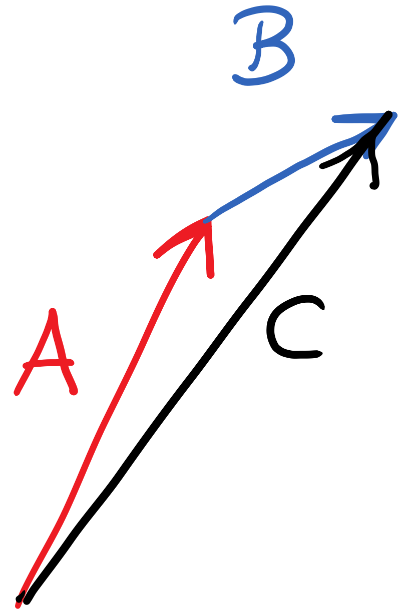


$$D_x = +D\sin\theta = +5km(0.8) = +4km$$

$$D_y = -D\cos\theta = -5km(0.6) = -3km$$

$$\vec{D} = +4km\,\hat{i} + (-3km)\,\hat{j}$$

Vector addition - graphically



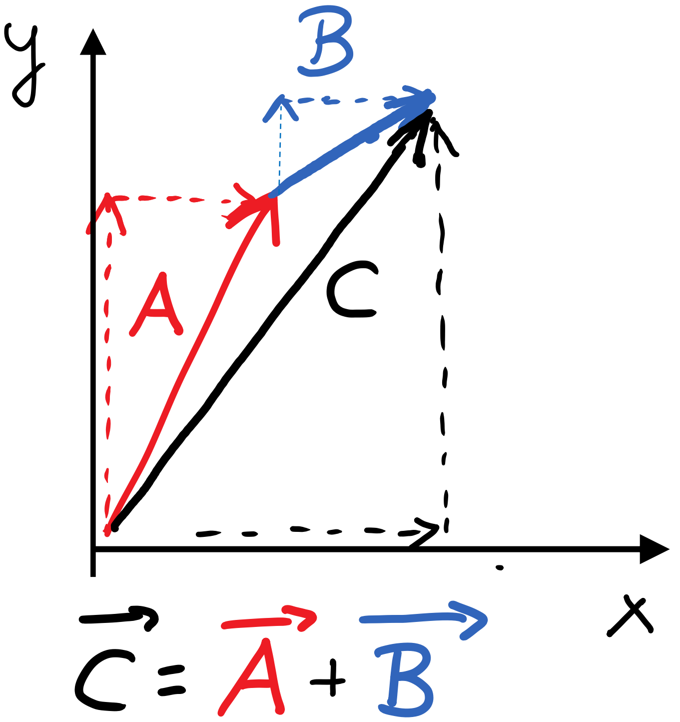
$$\vec{C} = \vec{A} + \vec{B}$$

Vector addition in components

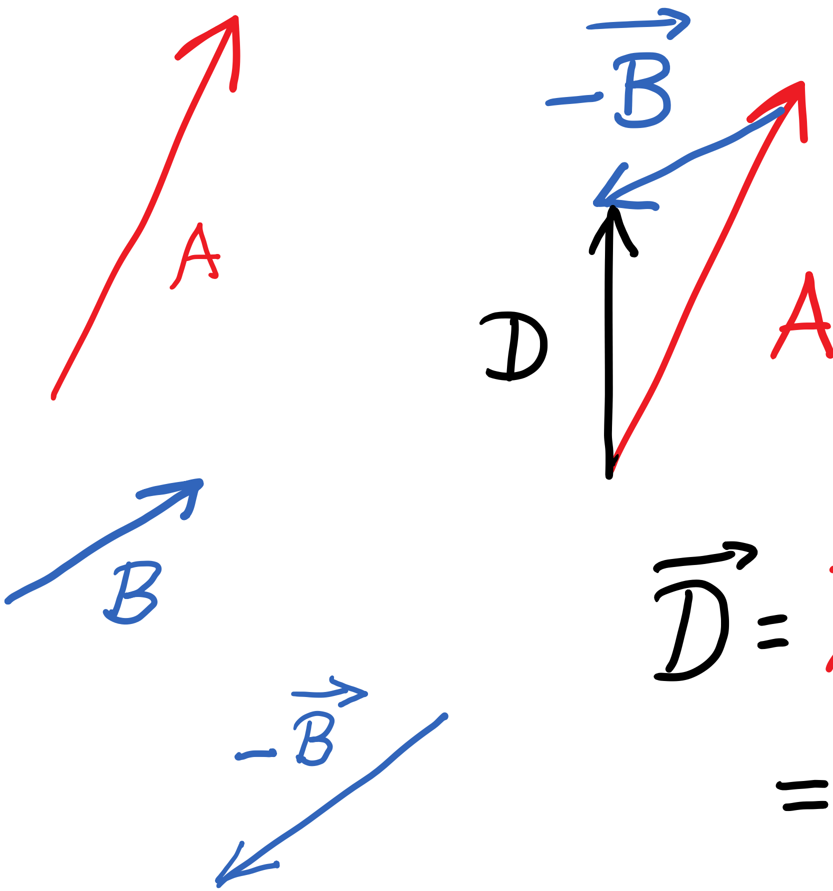
$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$$
$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath}$$

$$\vec{C} = \left(A_x\hat{\imath} + A_y\hat{\jmath}\right) + (B_x\hat{\imath} + B_y\hat{\jmath})$$

$$\vec{C} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath}$$
$$C_x = A_x + B_x$$
$$C_y = A_y + B_y$$



Vector subtraction - graphically



$$\vec{D} = \vec{A} - \vec{B}$$
$$= \vec{A} + (-\vec{B})$$

Vector subtraction in components

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$$

$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath}$$

$$\vec{D} = \vec{A} - \vec{B}$$

$$\vec{D} = \left(A_x\hat{\imath} + A_y\hat{\jmath}\right) - (B_x\hat{\imath} + B_y\hat{\jmath})$$

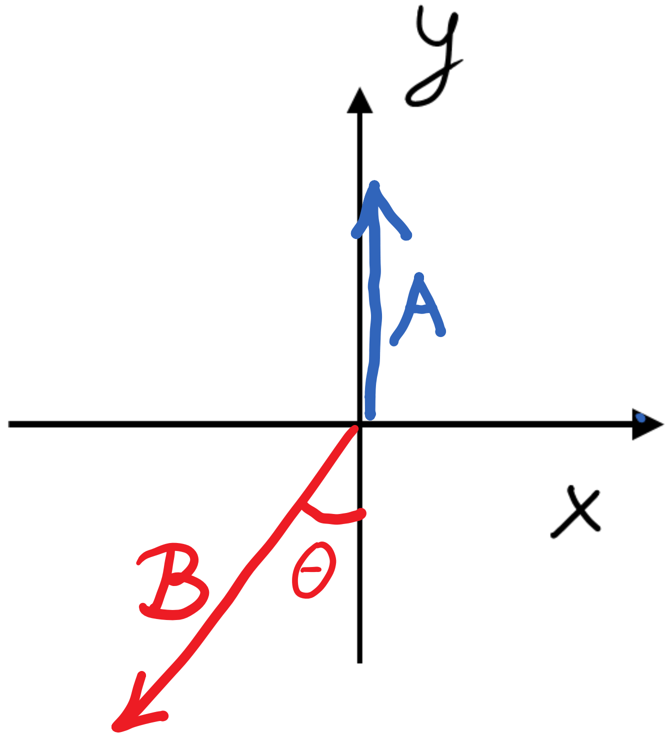
$$\vec{D} = (A_x - B_x)\hat{\imath} + (A_y - B_y)\hat{\jmath}$$

$$D_x = A_x - B_x$$

$$D_y = A_y - B_y$$

Example

Express the vector $\vec{C} = \vec{A} + \vec{B}$ in unit vector notation in terms of A, B, and θ.



Velocity

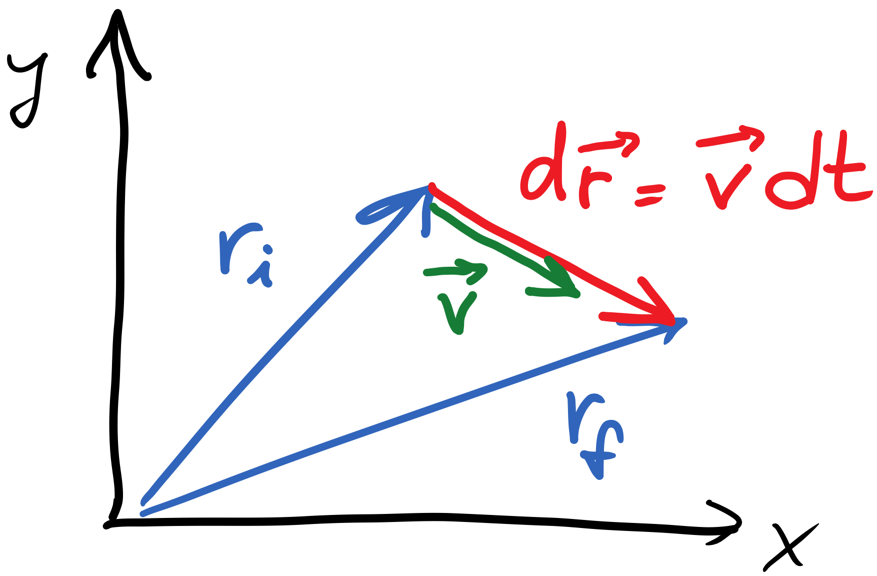
Position vector $\vec{r} = r_x\hat{\imath} + r_y\hat{\jmath} = x\hat{\imath} + y\hat{\jmath}$

Average velocity $\vec{v}_{av} = \dfrac{\Delta\vec{r}}{\Delta t}$

Instantaneous velocity: $\vec{v} = \dfrac{d\vec{r}}{dt}$

$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath}$$

$$v_x = \frac{dx}{dt} \quad , \quad v_y = \frac{dy}{dt}$$



Small change of position vector in the direction of velocity vector

$$\vec{r}_f = \vec{r}_i + \vec{v}dt$$

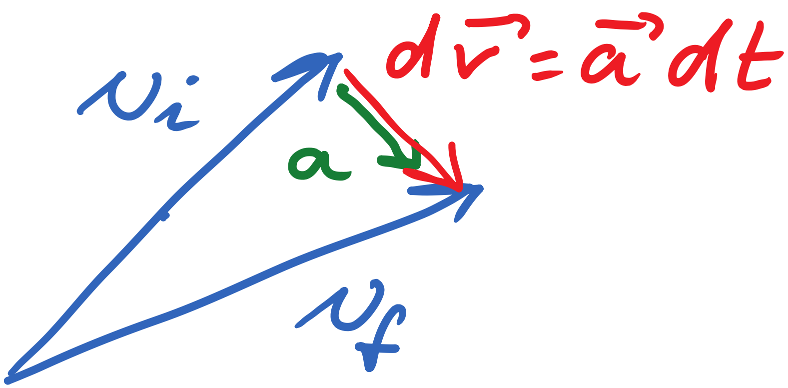
Acceleration

Particle has velocity vector $\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath}$

Acceleration: $\vec{a} = \frac{d\vec{v}}{dt}$

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} = \frac{dv_x}{dt}\hat{\imath} + \frac{dv_y}{dt}\hat{\jmath}$$

$$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2} \quad , \quad a_y = \frac{dv_y}{dt} = \frac{d^2y}{dt^2}$$



Small change in velocity vector occurs in the direction of the acceleration vector

$$\vec{v}_f = \vec{v}_i + \vec{a}dt$$

Acceleration changes velocity, i.e. speed and direction of motion.

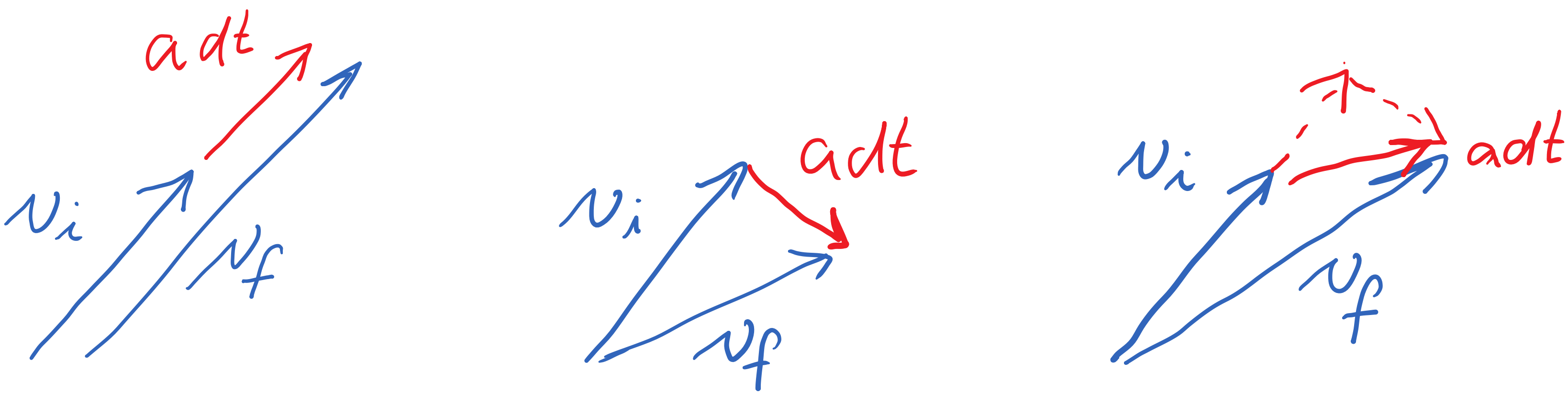
Effect of acceleration components

Components of acceleration parallel and perpendicular to velocity have different effects.

$$d\vec{v} = \vec{a}dt$$

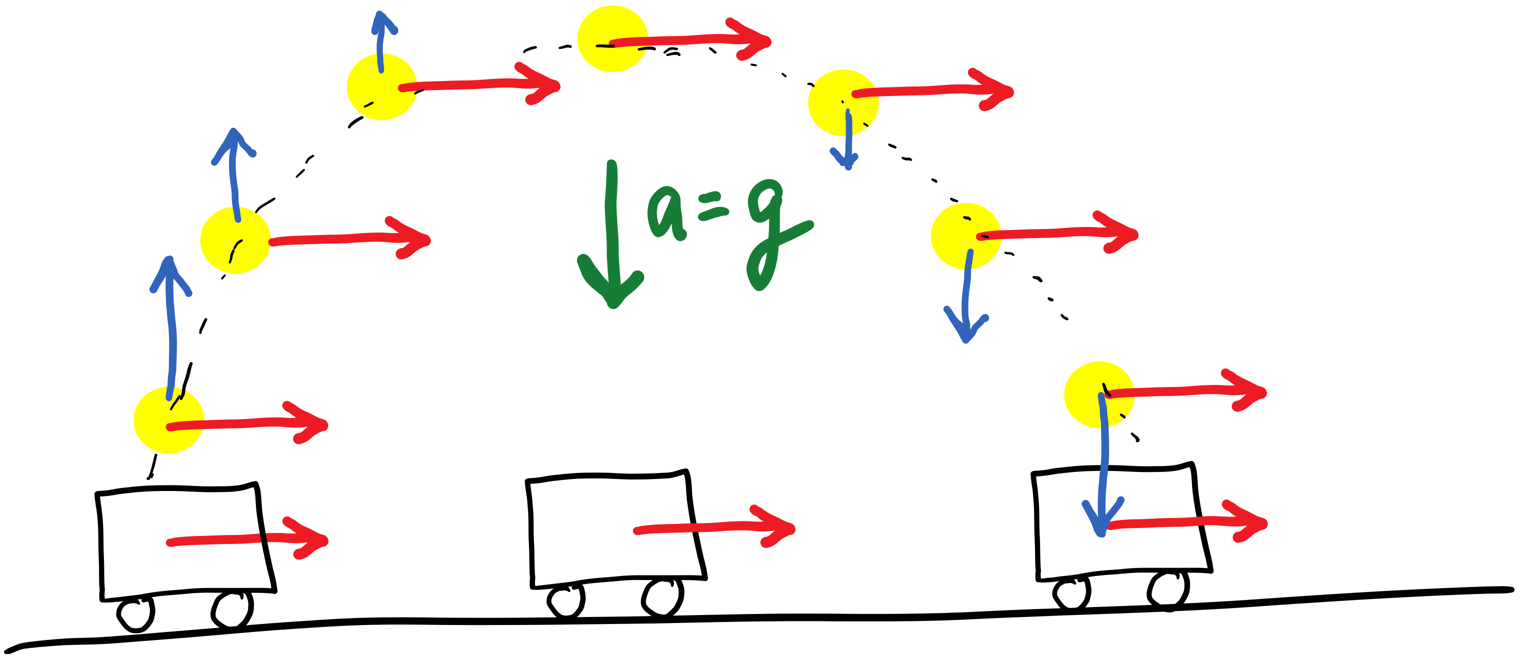
$a_{||}$ causes change in magnitude of velocity vector (speed)

$a_{\perp}$ causes change in direction



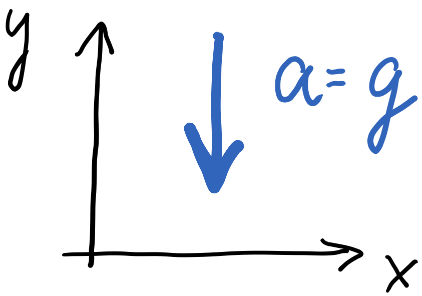
Demonstrations

- Vertical launch of ball from traveling car
- Simultaneously dropped and horizontally launched balls
  (see HW problem #3)


a=g

Projectile Motion

If only gravity acts on an object (free fall), then acceleration is a constant vector of magnitude g, directed down.



$$a_x = 0$$

$$a_y = -g$$

Effect on velocity:

$$v_x = v_{0x} + a_x t = v_{0x}$$

$$v_y = v_{0y} + a_y t = v_{0y} - gt$$

NOT starting equations