Lecture 4: Motion in two dimensions

- Equations for 2-d kinematics at constant acceleration
- Projectile motion
- Problem Solving

Kinematics equations

For constant acceleration:

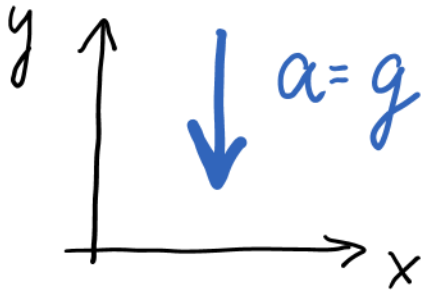
$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$
$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

$$v_x = v_{0x} + a_x t$$
$$v_y = v_{0y} + a_y t$$

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$$
$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

Projectile Motion

If only gravity acts on an object (free fall), then acceleration is a constant vector of magnitude g, directed down.



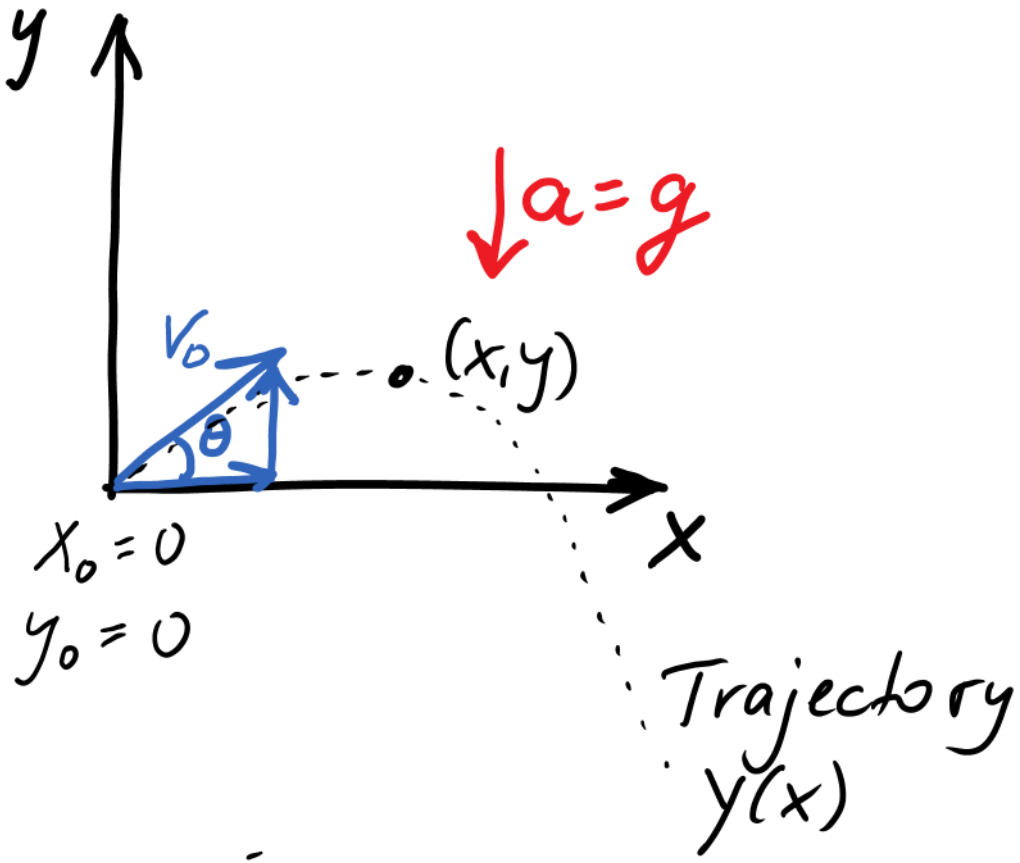
$$a_x = 0$$
$$a_y = -g$$

$$v_x = v_{0x} + a_x t = v_{0x}$$
$$v_y = v_{0y} + a_y t = v_{0y} - gt$$

Projectile motion: Simulation

Link to Water Fendt Projectile Simulation

Free-fall trajectory

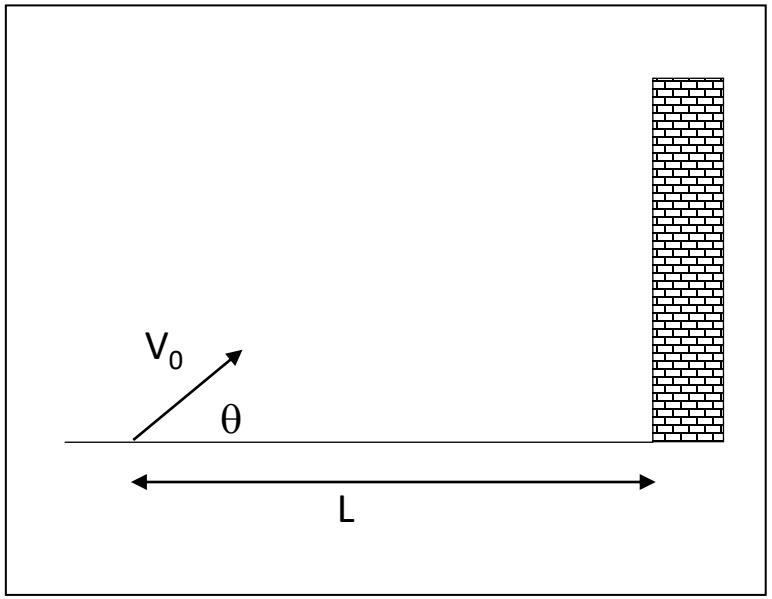


$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

Example

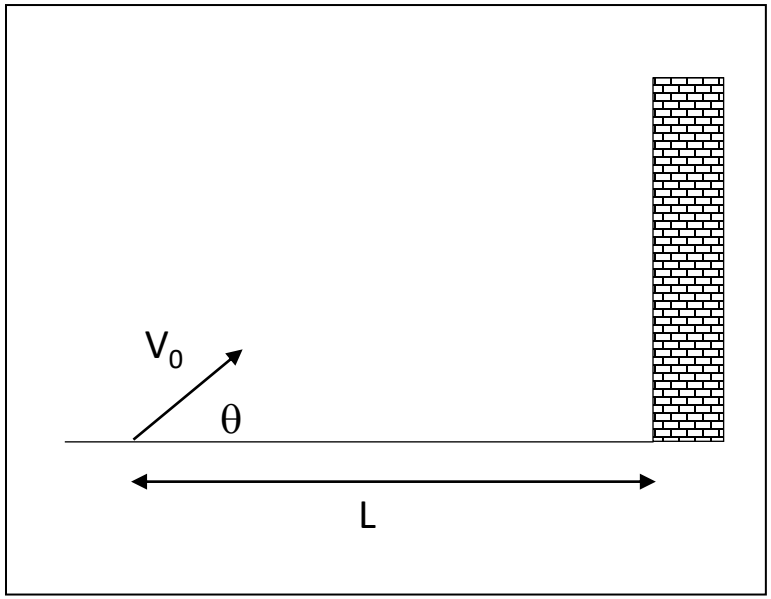
A child kicks a soccer ball from the ground level with an initial speed V_0 at an angle θ with respect to the horizontal. The ball hits a wall a distance *L* away.



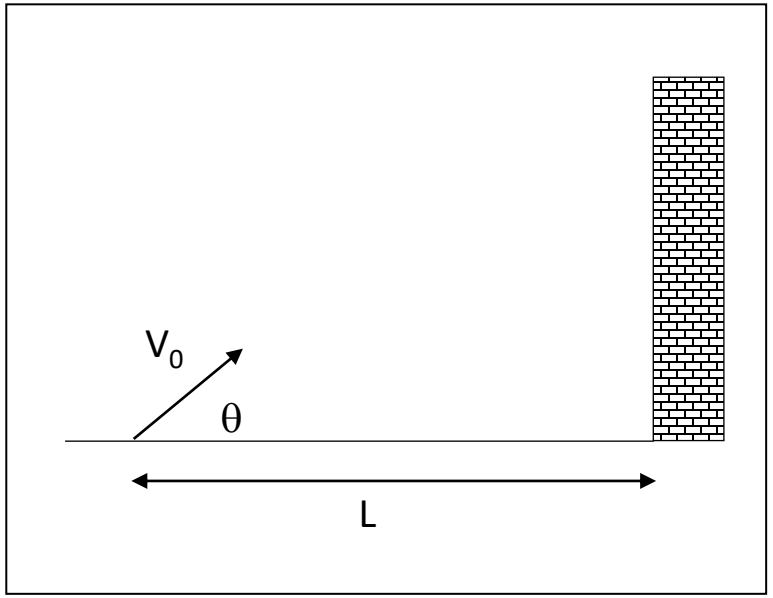
a) Complete the diagram with all information necessary to solve the parts below.

b) Derive a symbolic expression for the time it takes the ball to reach the wall.

c) Derive a symbolic expression for the height *H* at which the ball hits the wall.


V_0
θ
L

d) The ball reaches its highest point before hitting the wall. Find the maximum height above the ground.



Example: Range of a Projectile

Demo: The hunter and the monkey