Lecture 10: Work and kinetic energy

- Kinetic energy
- Vector dot product (scalar product)
- Definition of work done by a force on an object
- Work-kinetic-energy theorem

Kinetic energy

$K = \frac{1}{2} m v^2$

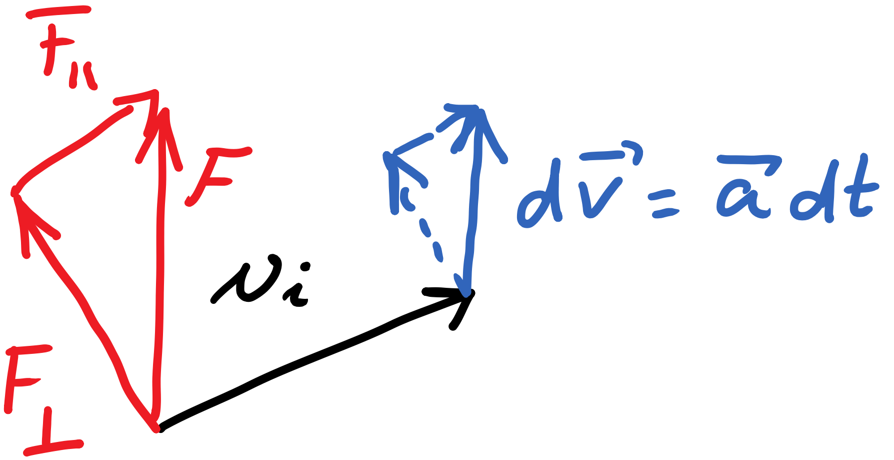
Kinetic energy is a scalar

It does not have components!

Unit: $kg \left(\frac{m}{s}\right)^2 = \left(kg \frac{m}{s^2}\right) m = Nm = J$ Joule

Effect of force components

Components of force parallel and perpendicular to velocity have different effects.



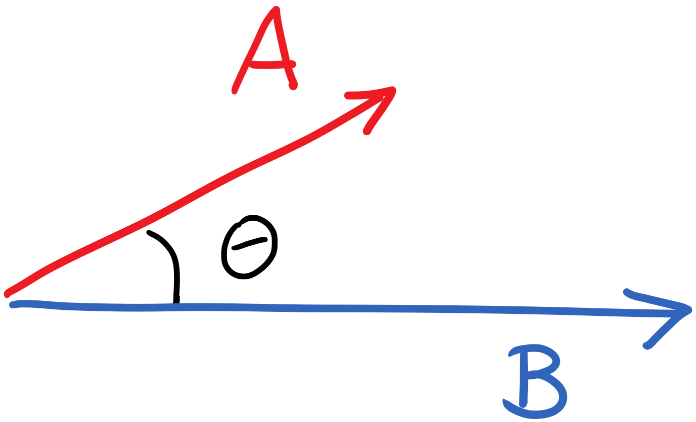
$F_{||}$ causes change in magnitude of velocity vector (speed)
⟹ changes kinetic energy

$F_{\perp}$ causes change in direction
⟹ does not change kinetic energy

Dot product

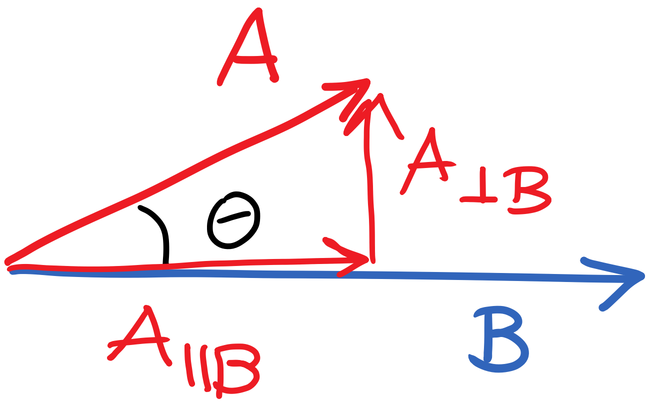
We need to find a way to determine how much of two vectors is parallel

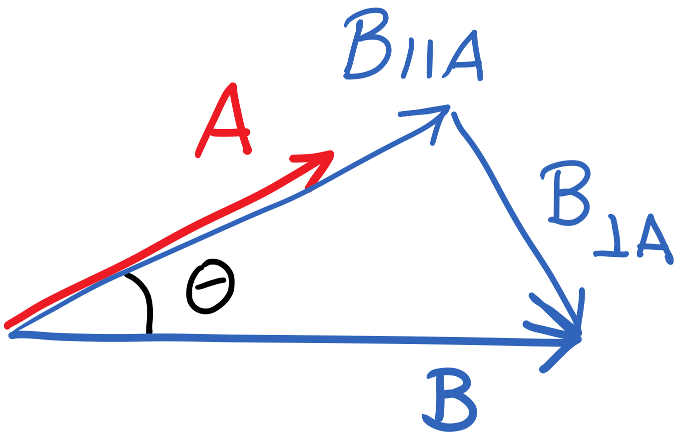
$$\vec{A} \cdot \vec{B} = AB\cos\theta$$ Vector dot product



The dot product is a scalar.

$$\vec{A} \cdot \vec{B} = AB\cos\theta = A_{\parallel B} B = A\, B_{\parallel A}$$

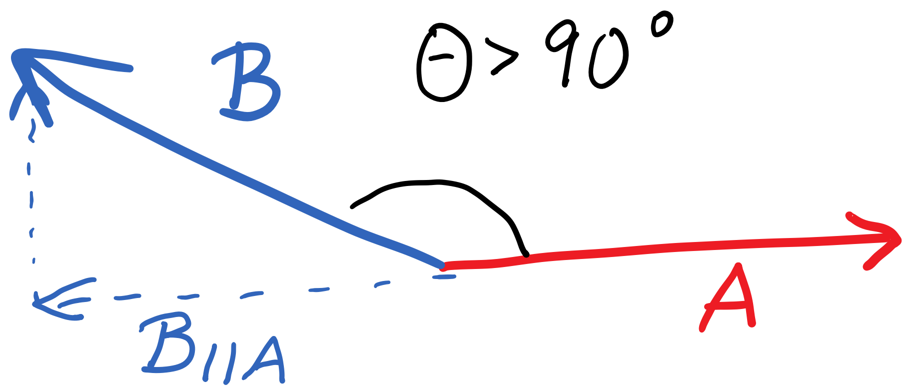




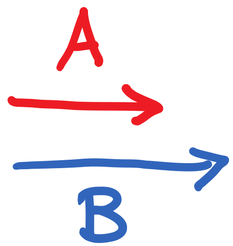
Dot product selects parallel component

What if θ > 90º?

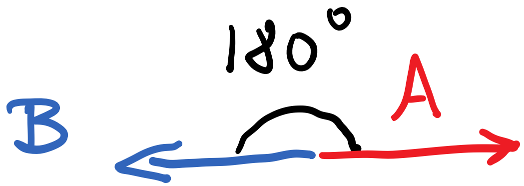
$$\vec{A}\cdot\vec{B} = AB\underbrace{\cos\theta}_{<0} = A\,\underbrace{B_{\parallel A}}_{<0}$$



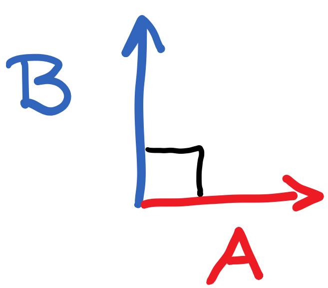
Special cases



$$\vec{A}\cdot\vec{B} = AB\cos 0^\circ = AB$$



$$\vec{A}\cdot\vec{B} = AB\cos 180^\circ = -AB$$



$$\vec{A}\cdot\vec{B} = AB\cos 90^\circ = 0$$

Properties of dot product

$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$ commutative

$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$ distributive

$\hat{\imath} \cdot \hat{\imath} = \hat{\jmath} \cdot \hat{\jmath} = \hat{k} \cdot \hat{k} = 1$ Same unit vectors

$\hat{\imath} \cdot \hat{\jmath} = \hat{\imath} \cdot \hat{k} = \hat{\jmath} \cdot \hat{k} = 0$ Orthogonal unit vectors

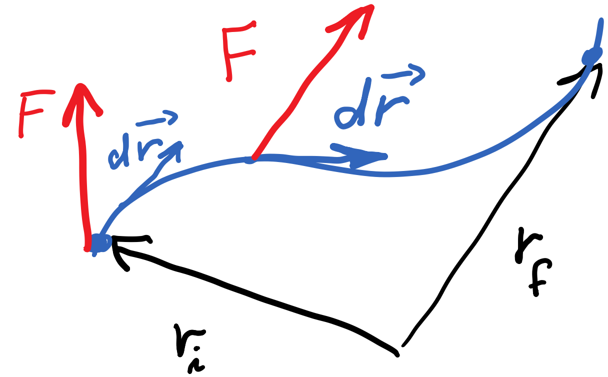
$\Longrightarrow$Dot product
in components:

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

Work

Work done on an object by a force $\vec{F}$ as the object moves along a path $\vec{r}(t)$ from initial to final position:

$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F}(\vec{r}) \cdot d\vec{r} = \int_{\vec{r}_i}^{\vec{r}_f} F_{\parallel} dr$$



Selects component of force parallel to path, i.e. parallel to velocity, which changes kinetic energy

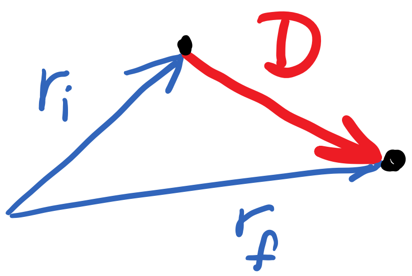
Force can vary in magnitude and direction along the path

Work by constant force

$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F}(\vec{r}) \cdot d\vec{r} = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r} = \vec{F} \cdot \int_{\vec{r}_i}^{\vec{r}_f} d\vec{r} = \vec{F} \cdot \left(\vec{r}_f - \vec{r}_i\right)$$

$$W = \vec{F} \cdot \overrightarrow{D}$$

$\overrightarrow{D} = \vec{r}_f - \vec{r}_i$ total displacement vector

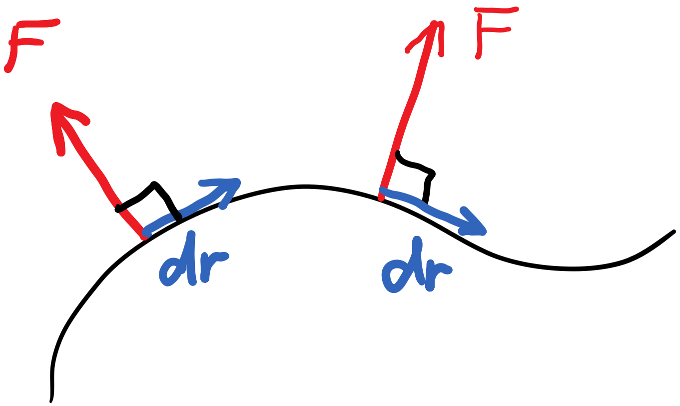


Caution: Force must be constant in magnitude and direction

Force perpendicular to path

If force is perpendicular to the path at every point*

$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F}(\vec{r}) \cdot d\vec{r} = \int_{\vec{r}_i}^{\vec{r}_f} F(\vec{r}) \underbrace{\cos 90°}_{0} dr = 0$$

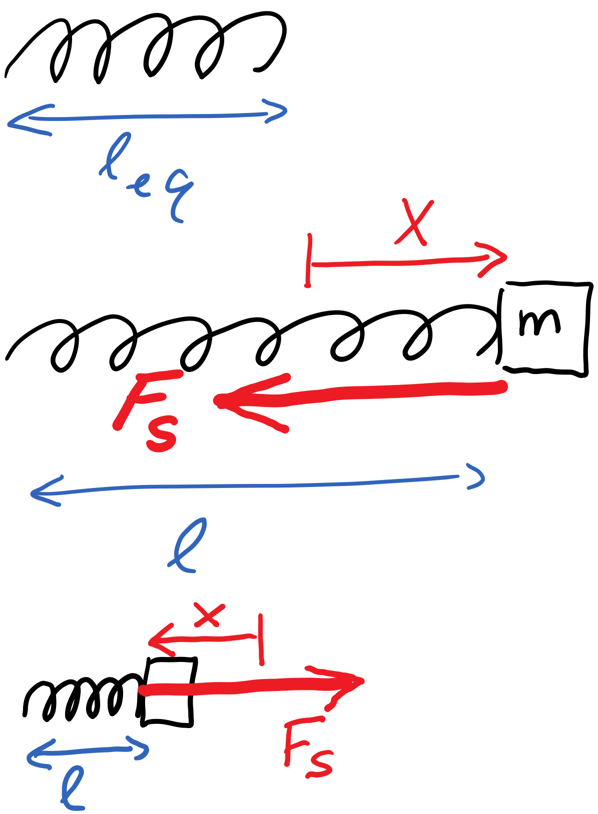


*whether constant in magnitude or not

Work done **by** a spring

Spring force: $F_{S\,x} = -kx$

$x = l - l_{eq}$
stretch or compression
k force constant



$$W_s = \int_{x_i}^{x_f} \vec{F}_S \cdot d\vec{x} = \int_{x_i}^{x_f} F_S dx \underbrace{\cos 180°}_{-1}$$

$$= \int_{x_i}^{x_f} -kx dx = -\frac{1}{2}k\left(x_f^2 - x_i^2\right)$$

Net Work

Work done by the net force on object as it moves along path.
But also (and easier to calculate):

$$W_{net} = \Sigma W_n$$

Sum of work done by all individual forces

Work-Kinetic Energy-Theorem

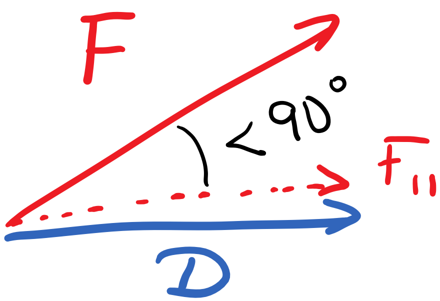
Work = effect of force component parallel to velocity
$\Longrightarrow$ changes kinetic energy

$$(W_{net})_{i\longrightarrow f} = \Delta K = K_f - K_i = ½mv_f^2 - ½mv_i^2$$

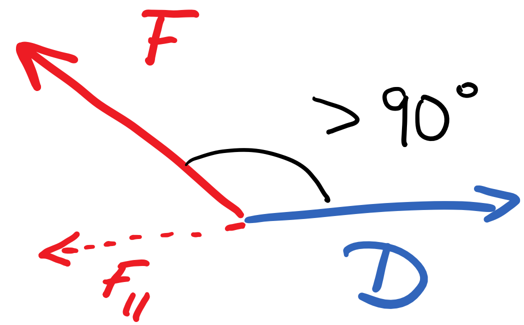
Sign of work

$$(W_{net})_{i\to f} = \Delta K = K_f - K_i = ½mv_f^2 - ½mv_i^2$$

Consider one single force acting on object



W positive
$\Delta K > 0,\ v_f > v_i$
speeds up



W negative
$\Delta K < 0,\ v_f < v_i$
slows down

Example

A block of mass M is pulled by a force of magnitude P directed at angle θ above the horizontal a distance D over a rough horizontal surface with coefficient of friction μ. Determine the change in the block's kinetic energy.