Lecture 14:
Fluid Dynamics

- Continuity
- Bernoulli’s Equation
- Viscosity

Ideal Fluid

- Steady (**laminar**) flow
- Incompressible
- Non-viscous

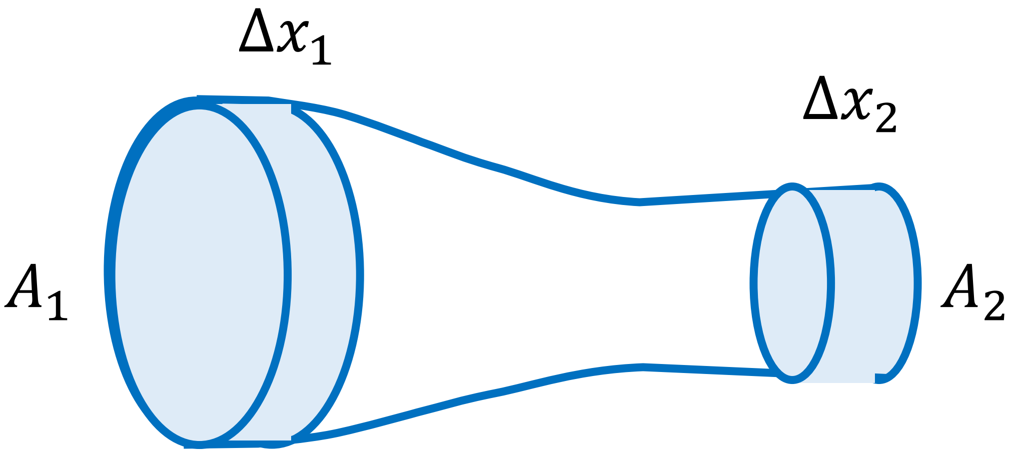
laminar

turbulent

Thermal convection plume rising from a candle in still air. The plume is initially laminar, but transition to turbulence occurs in the upper 1/3 of the image. Credit: Dr. Gary Settles

Continuity

"What goes in must come out"

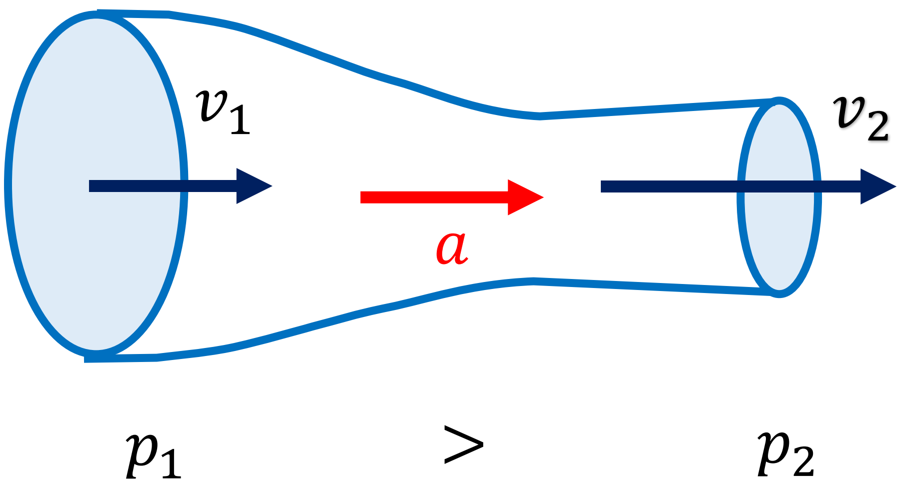


Volume flow rate: $\frac{\Delta V}{\Delta t} = A_1 \frac{\Delta x_1}{\Delta t} = A_2 \frac{\Delta x_2}{\Delta t}$

$$\frac{\Delta V}{\Delta t} = Av = constant$$

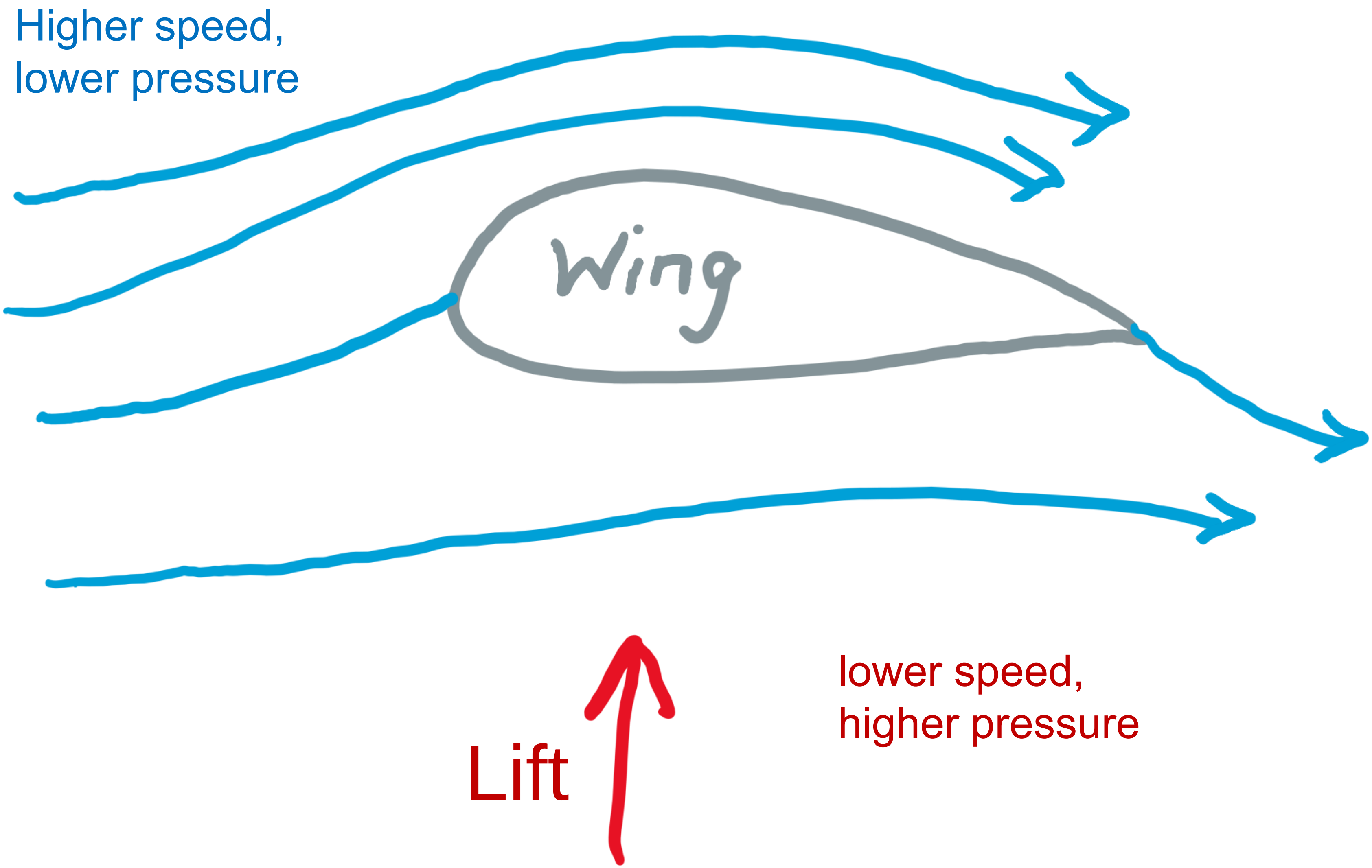
Bernoulli's Principle

Ideal fluid: no friction.
Fluid is accelerated by pressure difference.
$Av = constant$: large A, small v –small A, large v
Force to accelerate the fluid is provided by a difference in pressure.

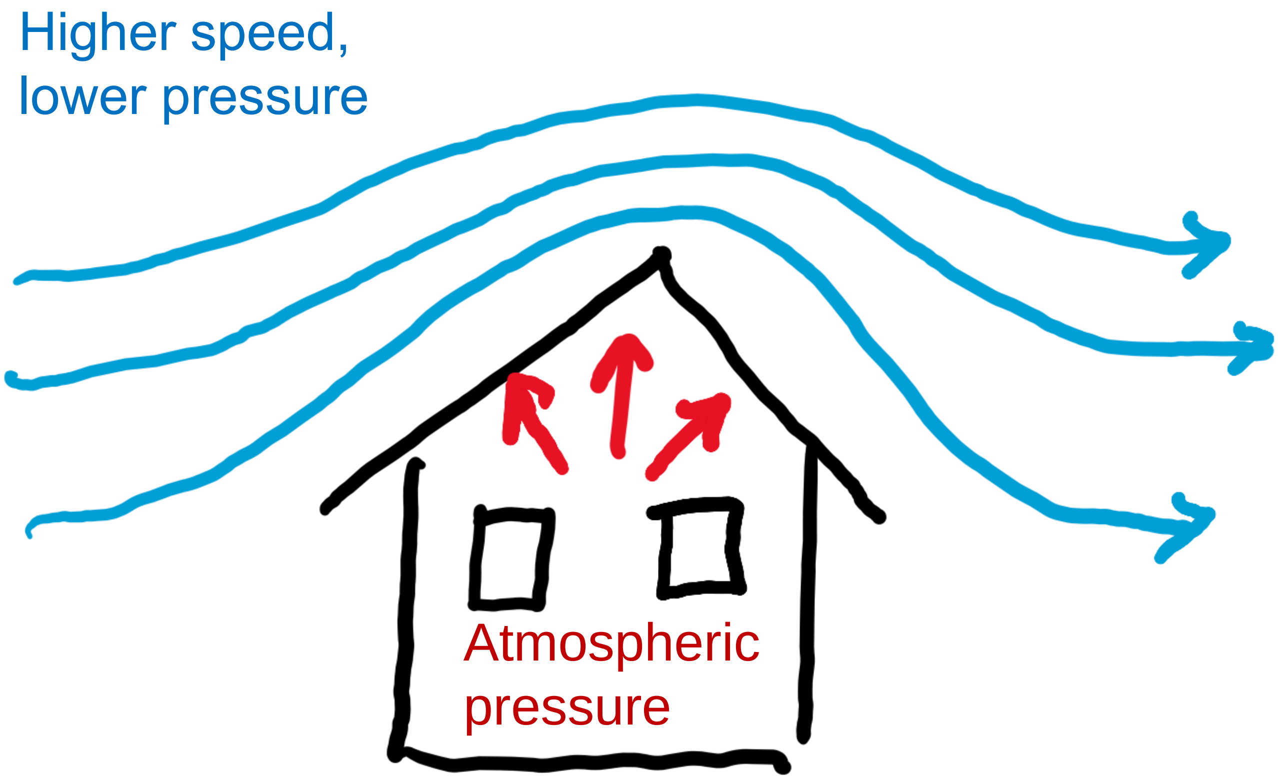


As fluid speed increases, pressure decreases.

Airplane wing

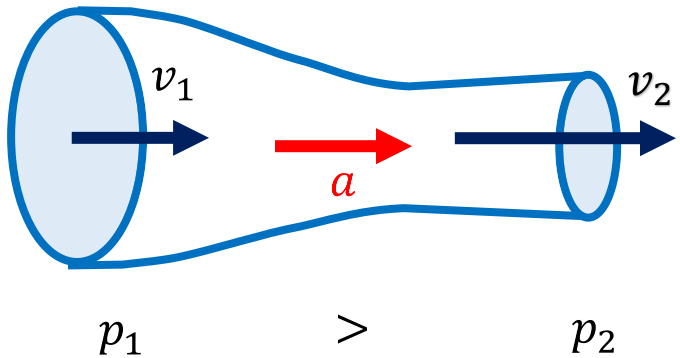


High wind lifts roof off house



Applications for Bernoulli’s Principle

As fluid speed increases, pressure decreases.

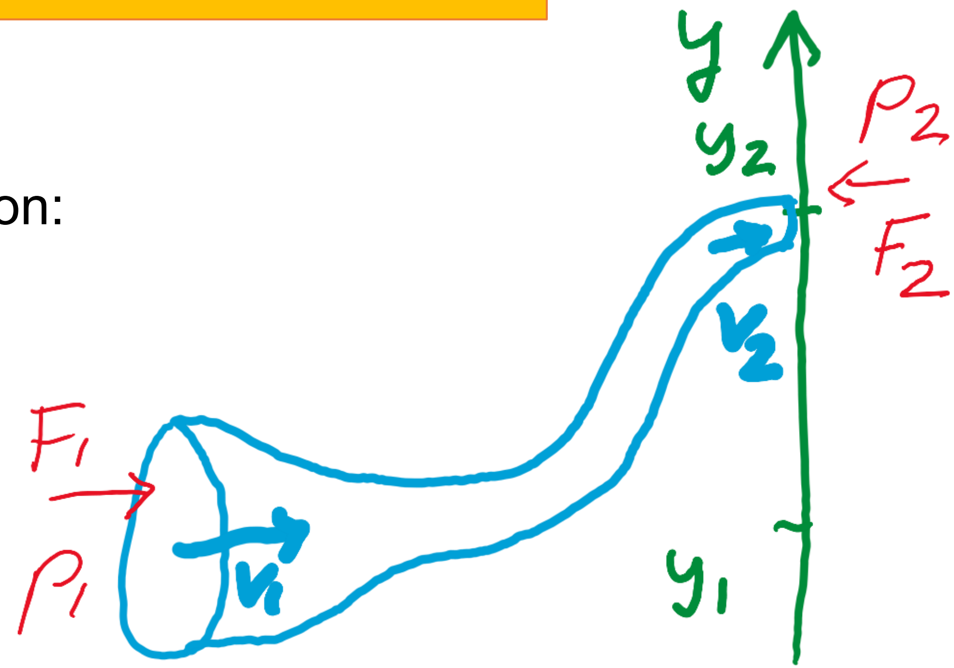


- ✈️ Air over airplane wing → Lift
- 🧪 Venturi tube (narrow pipe section) → Pressure drop
- 🌬️ Wind over a roof → Suction effect
- 🌫️ Pump spray → Suction draws fluid upward

Bernoulli's Equation

Application of energy conservation:

$$E_f - E_i = W_{other}$$



$$½mv_f^{\,2} + mgy_f - ½mv_i^{\,2} - mgy_i = F_1 d_1 - F_2 d_2$$

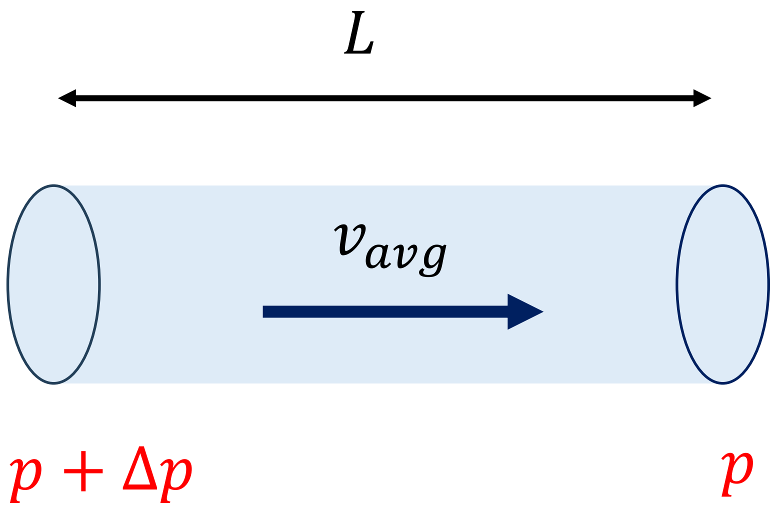
$$½mv_2^{\,2} + mgy_2 - ½mv_1^{\,2} - mgy_1 = p_1 A_1 d_1 - p_2 A_2 d_2 = p_1 V - p_2 V$$

$$p_1 + \frac{1}{2}\rho\, v_1^{\,2} + \rho\, g\, y_1 = p_2 + \frac{1}{2}\rho\, v_2^{\,2} + \rho\, g\, y_2$$

Example: Water Tank with Hole

Viscous fluids

Resistance to flow: A viscous fluid needs a pressure difference between the ends of a tube to keep the fluid moving at constant speed.
Compare: Moving a block at constant speed along rough surface requires a force.



$$\Delta p = 8\pi\eta\frac{Lv_{avg}}{A}$$

η coefficient of viscosity

Unit: Pa·s

Viscosity

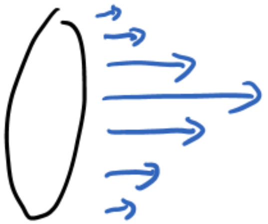
$$\Delta P = 8\pi\eta\frac{Lv_{avg}}{A} = \frac{8\eta Lv_{avg}}{r^2}$$

η coefficient of viscosity, unit: Pa·s

Ideal fluid:	0	No pressure difference needed
Water:	10^{-3} Pa·s	
Blood:	2.5×10^{-3} Pa·s	
Honey (40ºC)	20 Pa·s	
Honey (15ºC)	600 Pa·s	

Poiseuille's Equation

Viscous fluid: speed changes over cross section of tube, fastest in the center, outermost layer does not move at all



$$\Delta P = 8\pi\eta\frac{Lv_{avg}}{A} = \frac{8\eta Lv_{avg}}{r^2}$$

$$v_{avg} = \frac{r^2\Delta P}{8\eta L}$$

$$Q = \frac{\Delta V}{\Delta t} = v_{avg}A = \frac{\pi r^{4}\Delta P}{8\eta L}$$

Applications of Poiseuille's Equation

$$Q = \frac{\Delta V}{\Delta t} = v_{avg} A = \frac{\pi r^{4} \Delta P}{8 \eta L}$$

Blood Flow in Capillaries and Arteries

- Vessel radius strongly affects flow rate
- Basis for understanding blood pressure and vascular resistance

IV Drips and Catheters

- Predicts flow rate in tubing used for fluid delivery

Respiratory Physiology

- Airflow through small bronchioles: small reductions in airway radius drastically reduce flow (asthma, obstructive lung diseases)

Microfluids in chip design, inkjet printing, lubrication systems