Lecture 17: Linear momentum

- Define impulse and linear momentum
- Systems of particles
- Conservation of linear momentum
- Explosions and collisions

Cats playing with Newton's cradle

Linear Momentum

$$\vec{p} = m\vec{v}$$

Vector!

Newton's 2nd law: $\vec{F}_{net} = \frac{d\vec{p}}{dt}$

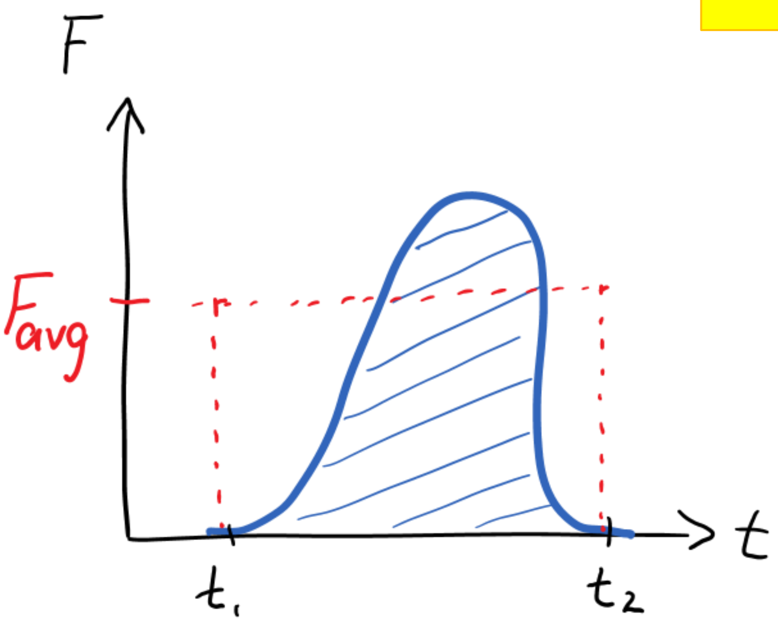
For constant m : $\vec{F}_{net} = \frac{d(m\vec{v})}{dt} = m\vec{a}$

Impulse

Impulse $\vec{J}$ delivered by force $\vec{F}$:

$$\vec{J} = \int_{t_i}^{t_f} \vec{F} dt$$

Vector!



$$\vec{J} = \vec{F}_{avg} \Delta t$$

Change in momentum and impulse

Newton's 2nd law: $\vec{F}_{net} = \frac{d\vec{p}}{dt}$

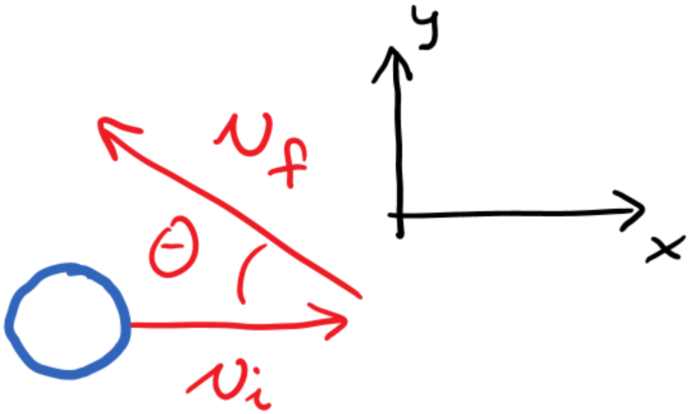
Integrate:

$$\int_{t_i}^{t_f} \vec{F}_{net} dt = \int_{t_i}^{t_f} \frac{d\vec{p}}{dt} dt$$

$$\vec{J}_{net} = \vec{p}_f - \vec{p}_i = \Delta\vec{p}$$

Example: kicking a ball

A soccer ball of mass m is moving with speed v_i in the positive x-direction. After being kicked by the player's foot, it moves with speed v_f at an angle θ with respect to the negative x-axis.



Calculate the impulse delivered to the ball by the player.

System of particles

$$\vec{P} = \sum_n \vec{p}_n = \sum_n m_n \vec{v}_n$$

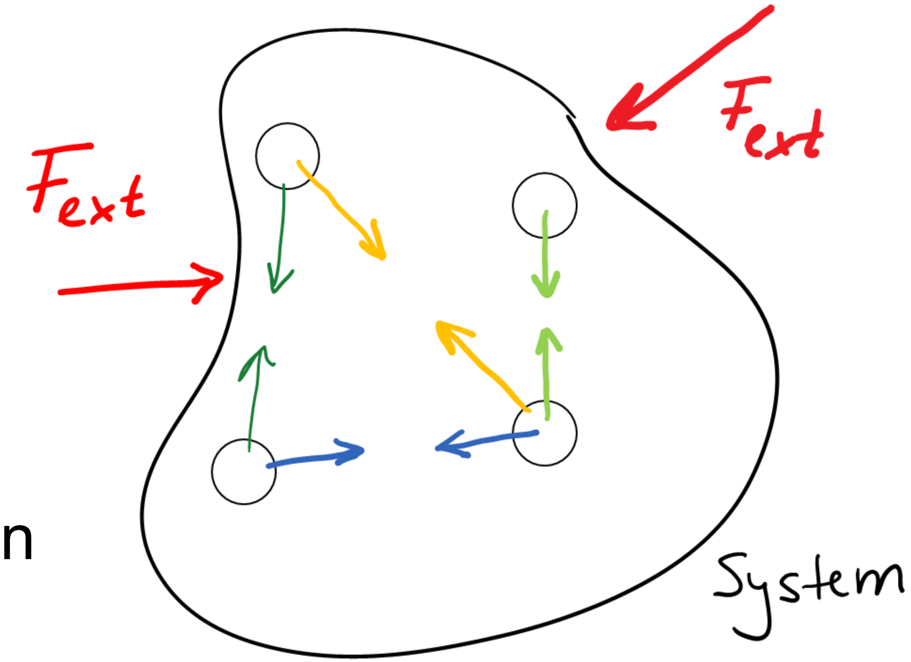
Linear momentum vector of system

Newton's 2nd law for system:

$$\vec{F}_{net} = \sum \vec{F} = \frac{d\vec{P}}{dt}$$

System of particles

$$\vec{F}_{net} = \sum \vec{F} = \frac{d\vec{P}}{dt}$$



Internal forces occur in action-reaction pairs, cancel.
Only external forces remain

$$\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt}$$

$$\vec{J}_{net\ ext} = \vec{P}_f - \vec{P}_i = \Delta\vec{P}$$

Conservation of linear momentum

If no external forces act:

$$\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt} = 0$$

$$\vec{P}_f = \vec{P}_i$$

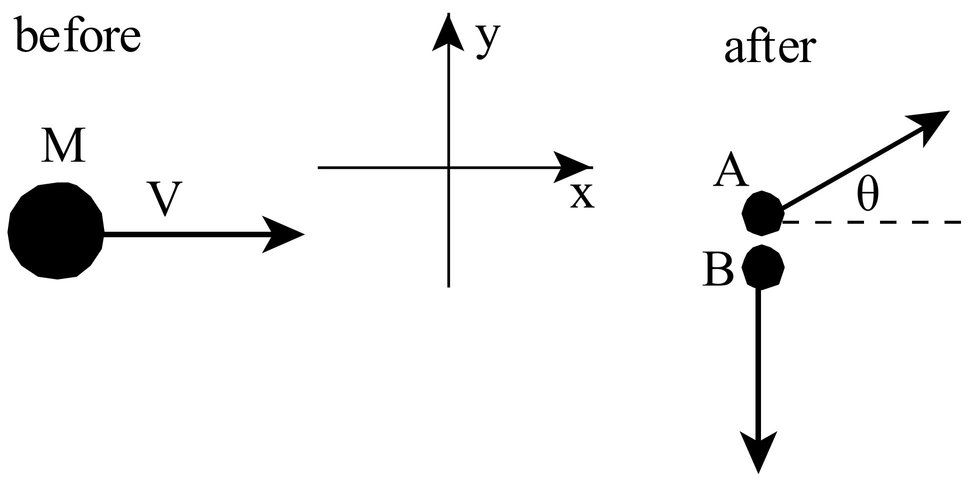
$$P_{fx} = P_{ix}$$
$$P_{fy} = P_{iy}$$

Example: explosions

Example: Explosion

A firecracker of mass *M* is traveling with speed *V* in the positive *x*-direction. It explodes into two fragments of equal mass. Fragment A moves away at an angle θ above the positive *x*-axis, as shown in the figure. Fragment B moves along the negative *y*-axis

Find the speeds of the fragments.



Recipe for Momentum Problems

1. Draw before and after sketch
2. Label masses and draw momentum/velocity vectors
3. Draw vector components
4. Starting equation.
5. Conservation of momentum if appropriate
6. Sum initial and final momenta
7. Express components
8. Solve symbolically

Short collisions

If collision happens in very short time:

$$\vec{J} = \int_{t_i}^{t_f} \vec{F} dt$$

- forces between colliding objects deliver dominating impulse
- impulse due to external forces negligible

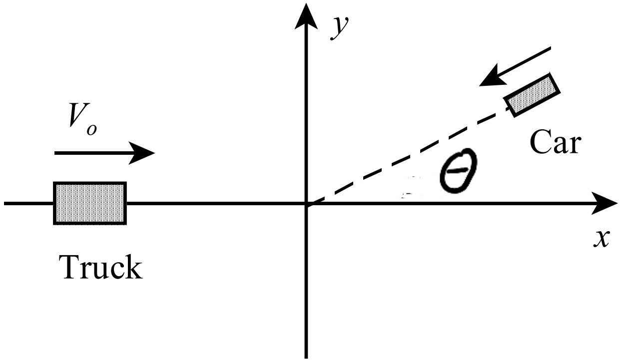
Example: car crash dominated by forces between the cars, effect of road friction negligible

$$\vec{J}_{ext} \approx 0 \longrightarrow \vec{P}_f \approx \vec{P}_i$$

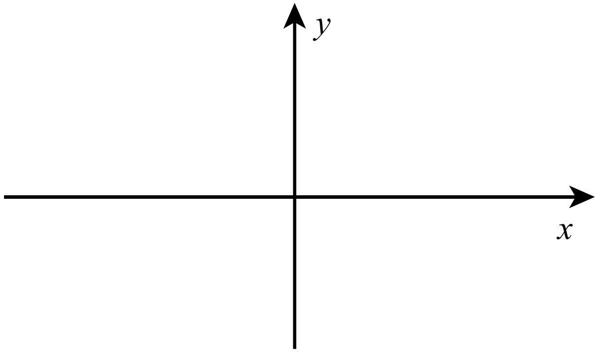
We can determine momenta right after the collision, before the wrecks skid on the pavement.

Example: Collision

A truck is moving with velocity V_o along the positive x-direction. It is struck by a car which had been moving towards it at an angle θ with respect to the x-axis. As a result of the collision, the car is brought to a stop, and the truck is moving in the negative y-direction. The truck is twice as heavy as the car.
Derive an expression for the speed V_f of the truck immediately after the collision



Before the collision



After the collision

Energy in collisions

In a quick collision:

- total linear momentum is conserved $\vec{P}_f = \vec{P}_i$
- total mechanical energy is usually **NOT** conserved

$$E_f \neq E_i$$

because non-conservative forces act (deforming metal)

➞ Inelastic collision

Perfectly inelastic: objects stick together after collision

Elastic collision: mechanical energy is conserved

Demo: Elastic and inelastic 1-D collisions on air track

Fractional change of kinetic energy

$$\frac{\Delta K}{K_i} = \frac{K_f - K_i}{K_i} = \frac{K_f}{K_i} - 1$$

Inelastic collisions: loss of kinetic energy (deformation etc)

Explosion: chemical energy is released and converted into kinetic energy