# Lecture 21:
# Torque

- Cross product
- Torque
- Relationship between torque and angular acceleration
- Problem solving

# What causes rotation?

Demo: bolt and wrench

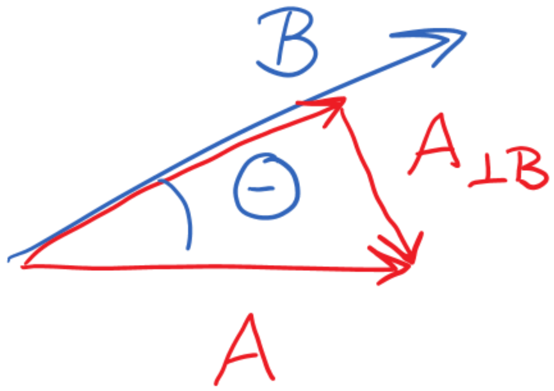
Need:

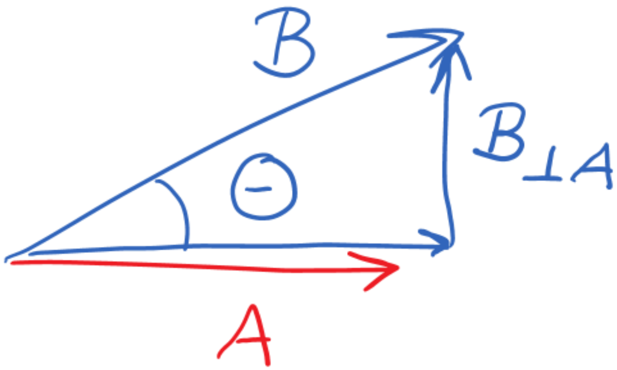
- Force
- Distance
- Perpendicular component

# Vector cross product: magnitude

$$\vec{A} \times \vec{B} = \vec{C}$$

$$C = AB\ sin\theta = A_{\perp B}B = AB_{\perp A}$$



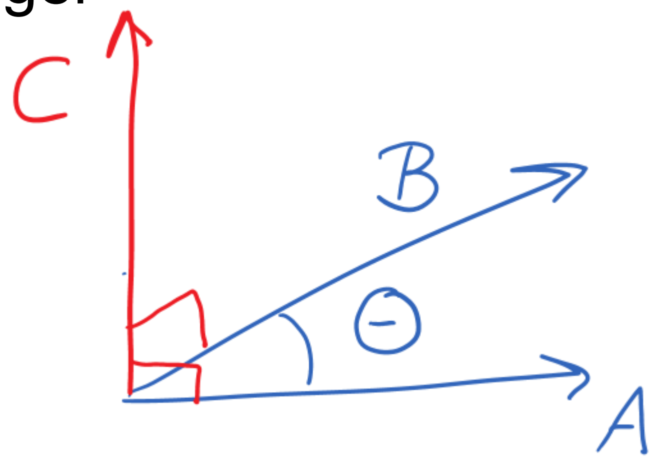



# Vector cross product: Direction

$$\vec{A} \times \vec{B} = \vec{C}$$

$\vec{C}$ is perpendicular to both $\vec{A}$ and $\vec{B}$

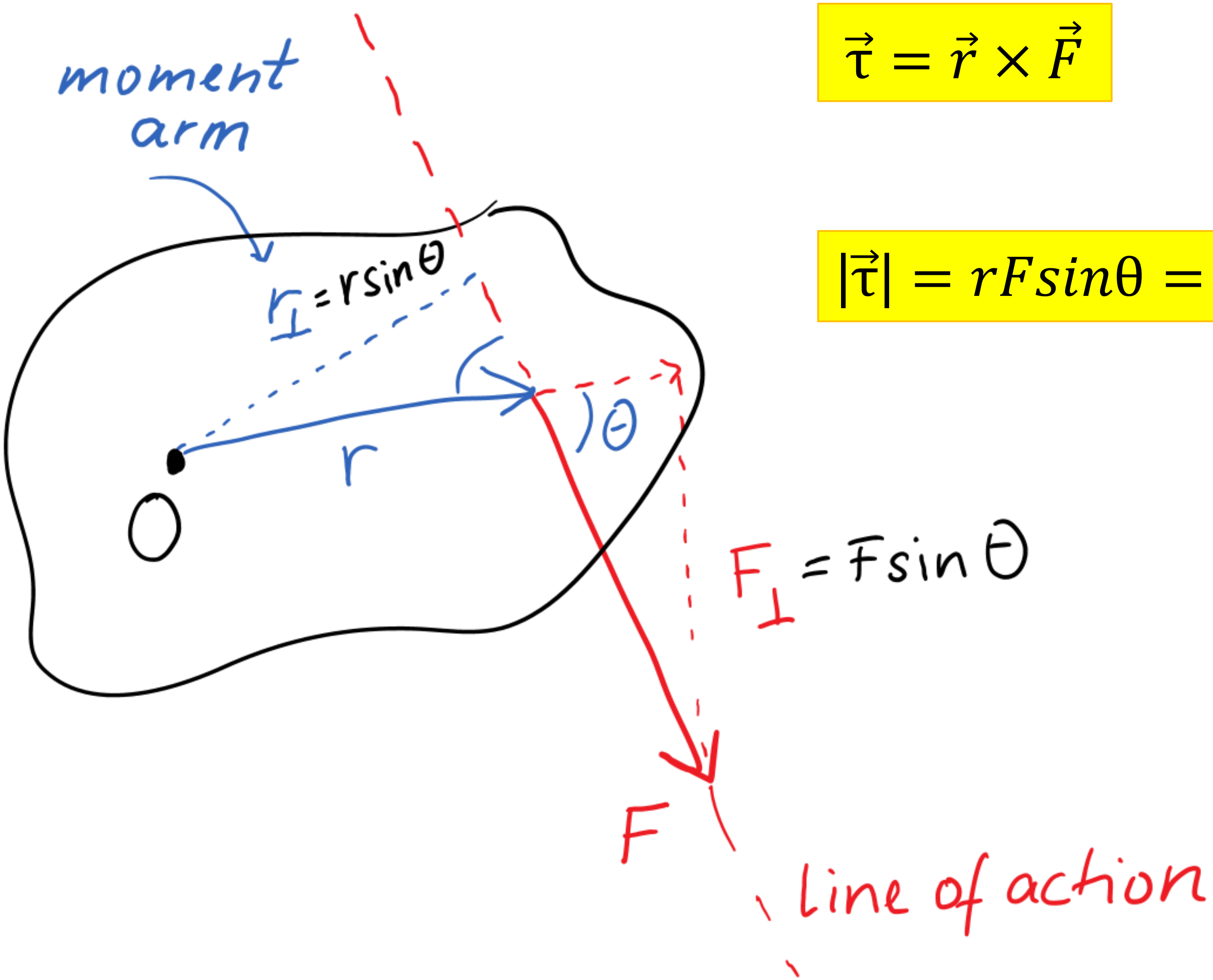
Direction: right hand rule
thumb × index finger = middle finger



# Torque

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$|\vec{\tau}| = rF\sin\theta = rF_{\perp} = r_{\perp}F$$



# Direction of torque

Right hand rule: $\vec{r} \times \vec{F} = \vec{\tau}$ $\quad\quad$ $\text{thumb} \times index = middle$

or easier:

If force tends to produce rotation in the positive $z$-direction, $\tau_z$ is *positive*:

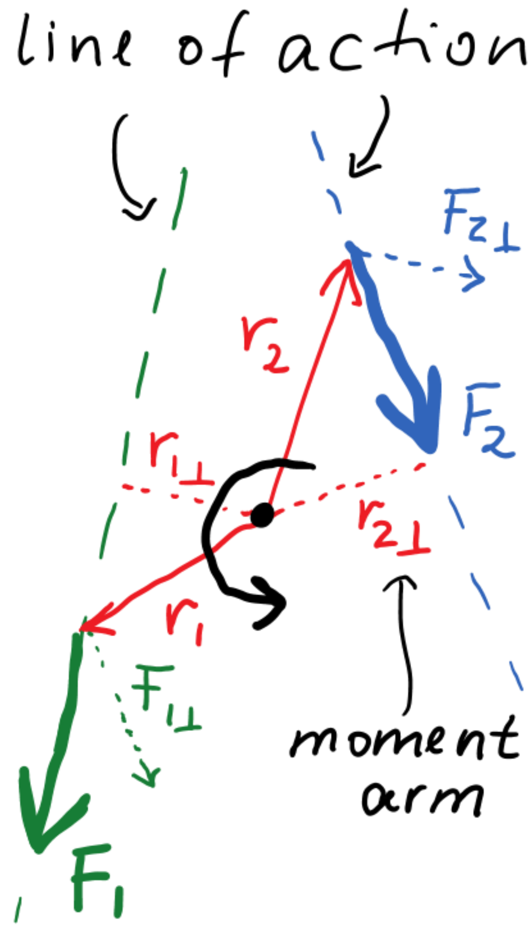
$$\tau_z \quad = \quad +r\,F\,\sin(\theta)$$

If force tends to produce rotation in the negative $z$-direction, $\tau_z$ is *negative*:

$$\tau_z \quad = \quad -\,r\,F\,\sin(\theta)$$

Indicate positive $z$-direction through curved arrow.

# Sign of torque component




$$\tau_{1z} = +F_1 r_{1\perp} = +F_{1\perp} r_1$$

$$\tau_{2z} = -F_2 r_{2\perp} = -F_{2\perp} r_2$$

Curved arrow indicates positive z-direction
Curl right-hand fingers; z-direction is thumb.

# Angular acceleration of rigid object

Rigid object that can rotate about $z$-axis.
$I_z$ moment of inertia about z-axis

$$\boxed{\sum \tau_z = I_z \alpha_z}$$

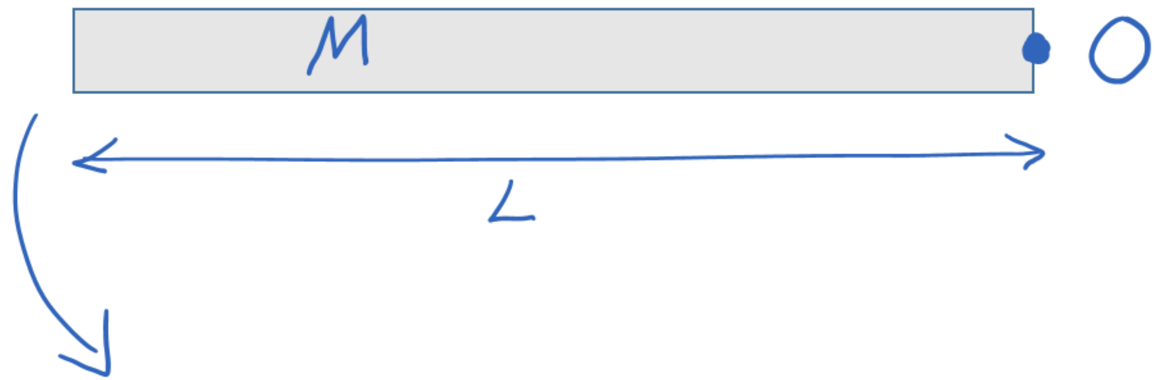
Compare to $\Sigma F_x = m a_x$

Begin with extended free-body diagram that shows forces and where they act on the object

# Example 1:

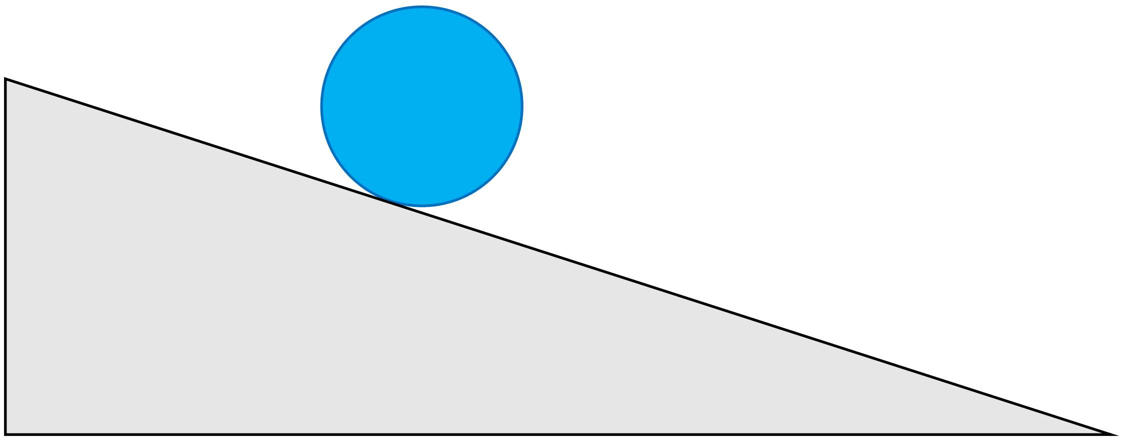
A uniform bar of length $L$ and mass $M$ can freely rotate about frictionless horizontal axis O at its end. The bar is initially in a horizontal position, is released from rest, and swings down under the influence of gravity. What is the initial angular acceleration of the bar just after it is released from rest?

$I_{bar} = \frac{1}{3} M L^2$ about O

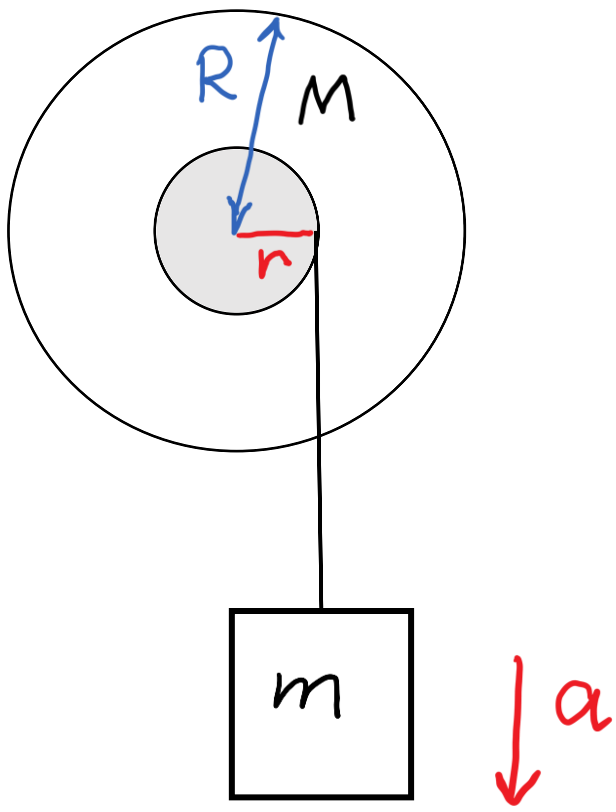


# Example 2: Rolling w/o slipping

An object of mass $M$, radius $R$ and moment of inertia $I$ is rolling without slipping down incline that makes an angle θ with the horizontal. Derive an expression for the object's linear acceleration.



# Example 3: Coupled objects




A small disk of radius $r$ is glued onto a large disk of radius $R$ that is mounted on a fixed axle through its center. The combined moment of inertia of the disks is $I$. A string is wrapped around the edge of the small disk, and a box of mass $m$ is tied to the end of the string. The string does not slip on the disk.
Find the acceleration of the box after it is released from rest.