Lecture 24:
Angular momentum

- Angular momentum of a point mass
- Angular momentum of a rigid rotating object
- Conservation of angular momentum

Translation vs rotation

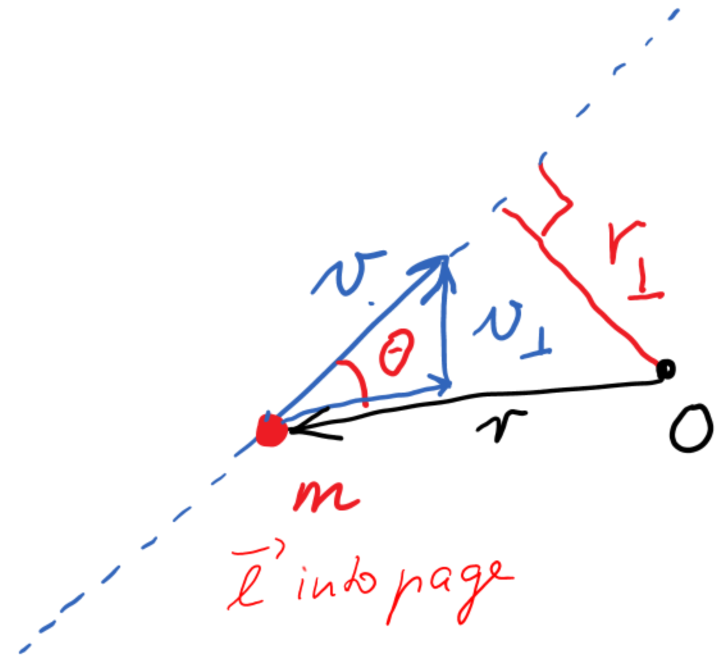
Linear momentum $\vec{p}$ is fundamental quantity for translation.
Forces change linear momentum.

Angular momentum $\vec{l}$ is fundamental quantity for rotation.
Torques change angular momentum.

$$\vec{l} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$$

Angular momentum of a particle

$$\vec{l} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$$



$$l = r_\perp mv = rmv_\perp = rmv\, sin\theta$$

Direction: right hand rule

$\vec{r} \times \vec{p} = \vec{l}$ perpendicular to $\vec{r}, \vec{p}$

$$thumb \times index\ finger = middle\ finger$$

Angular momentum of rigid object

For system of particles: $\vec{L} = \sum_n \vec{l}_n$

$$\sum_n l_n = \sum_n r_{n\perp} m_n v_n = \sum_n r_{n\perp} m_n (\omega_n \, r_{n\perp}) = \omega \sum_n m_n {r_{n\perp}}^2$$

$$\boxed{\vec{L} = I\vec{\omega}}$$

$\vec{L}$ is in the same direction as angular velocity $\vec{\omega}$ vector for rotations **about a symmetry axis only**.
This is not the case for rotation about other axes.

Angular momentum conservation

$$\sum \tau_z = \frac{dL_z}{dt} = \frac{d(I\omega_z)}{dt} = I\alpha_z$$

For system: $$\sum \vec{\tau} = \sum \vec{\tau}_{ext} = \frac{d\vec{L}}{dt}$$

If $$\boxed{\sum \vec{\tau}_{ext} = 0 \Longrightarrow \frac{d\vec{L}}{dt} = 0, \vec{L}_i = \vec{L}_f}$$

Compare: $\sum F_x = \frac{dP_x}{dt}$ If $\sum F_{ext,x} = 0, P_{ix} = P_{fx}$

Demo: Rotating Chair and Dumb Bells

$$\sum \vec{\tau}_{ext} = 0 \Longrightarrow \frac{d\vec{L}}{dt} = 0, \vec{L}_i = \vec{L}_f$$

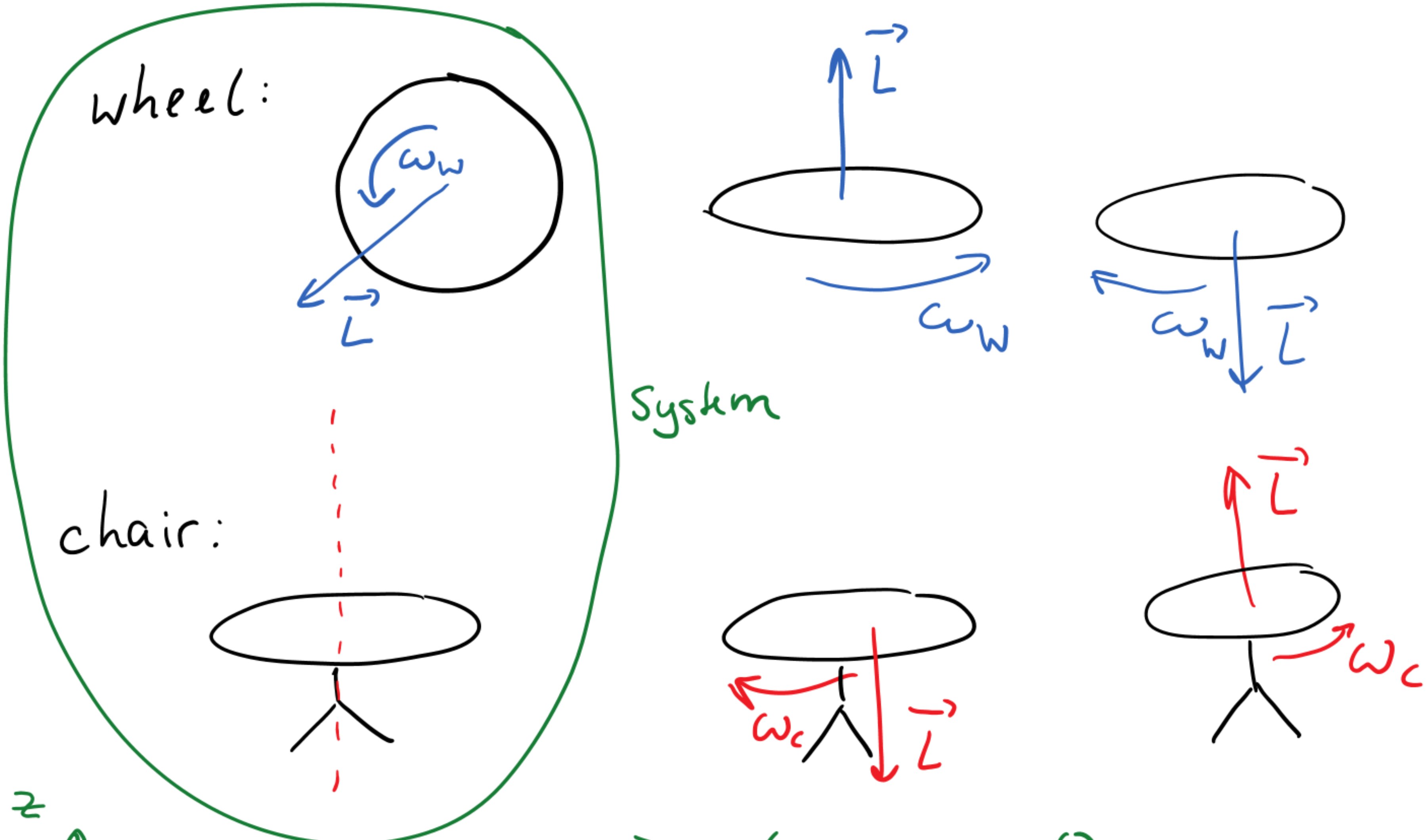
$$\boxed{I_i \omega_i = I_f \omega_f}$$

$$I = \sum_n m_n r_n^2$$

big I
small ω

small I
big ω

Demo: Rotating chair and Bicycle Wheel

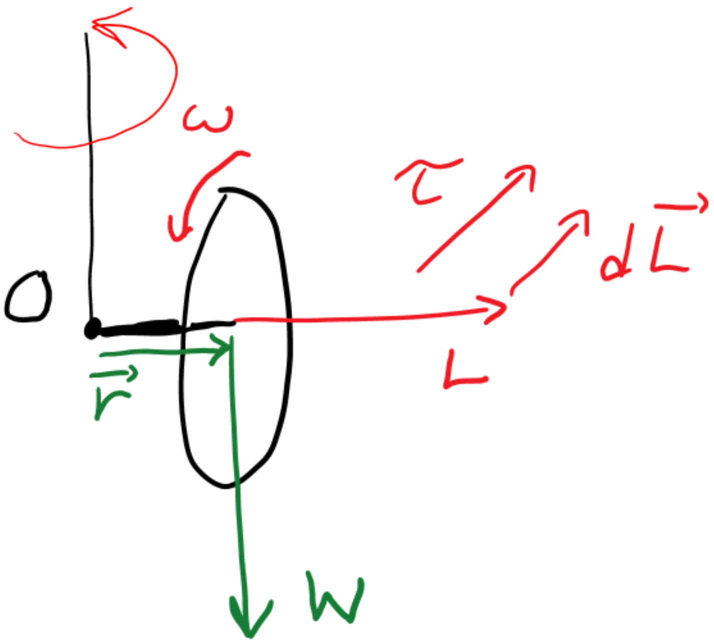


$$L_{z\,system} = 0 \implies L_{z\,system} = 0$$

Demo: Bicycle Wheel Gyroscope

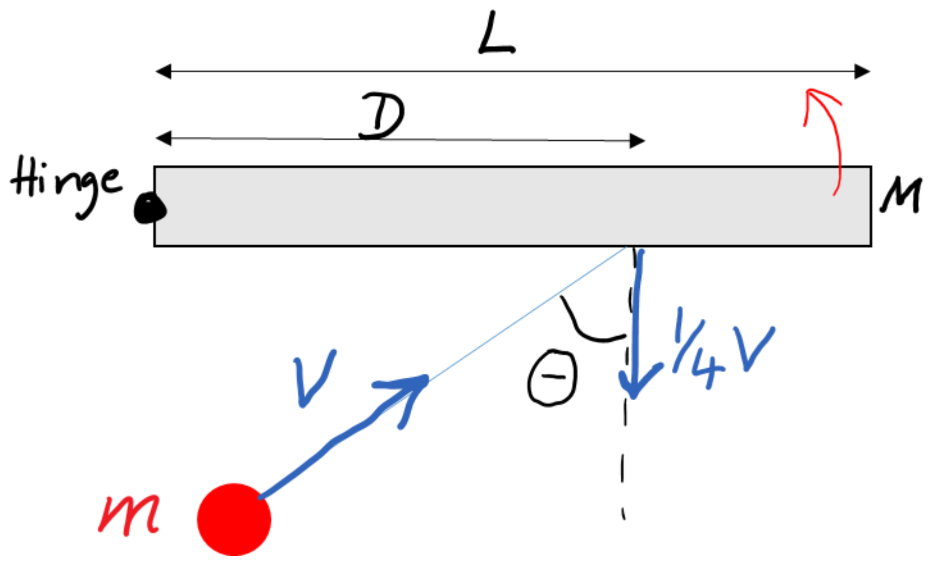
$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

$$d\vec{L} = \vec{\tau}\,\mathrm{d}t$$



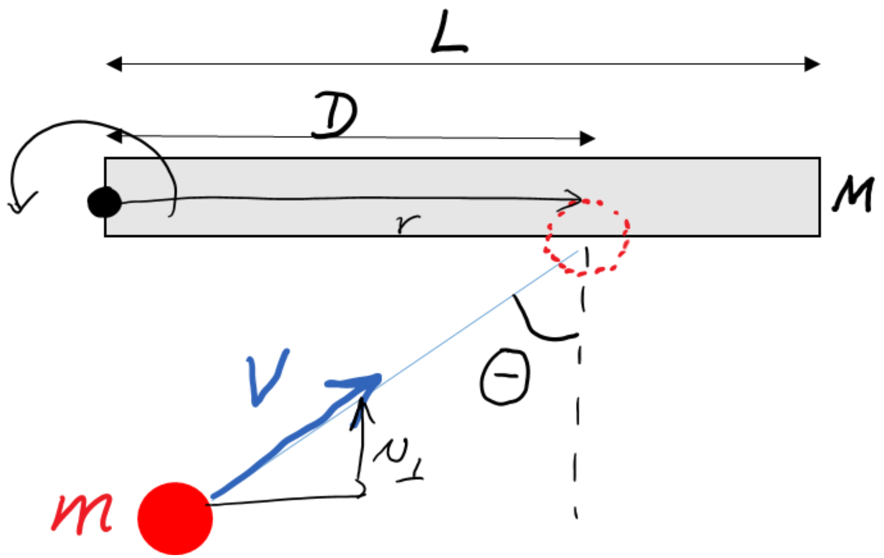
Example 1

A ball of mass m and speed V strikes a rod at angle θ and bounces off at a right angle with ¼ its original speed. What is the final angular speed of the rod after the collision?



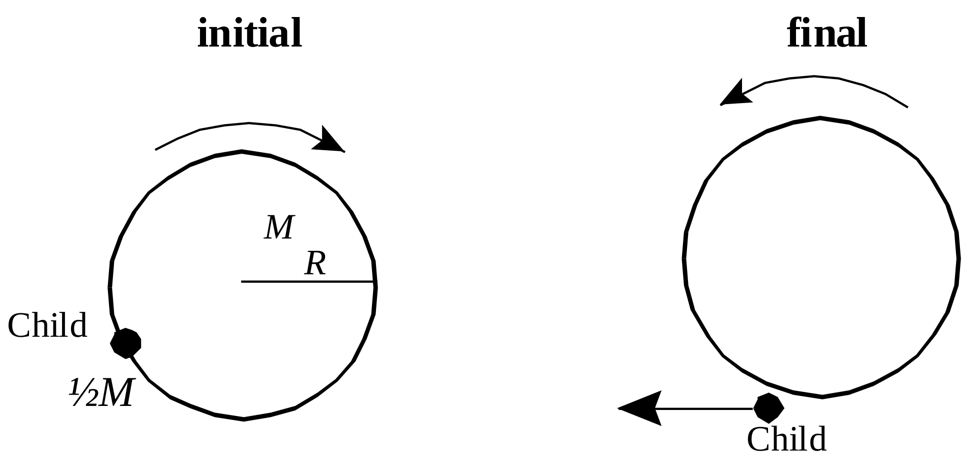
Example 2

A ball from example 1 is made of putty and sticks to the rod after the collision. What is the final angular speed of the rod with the ball stuck on?



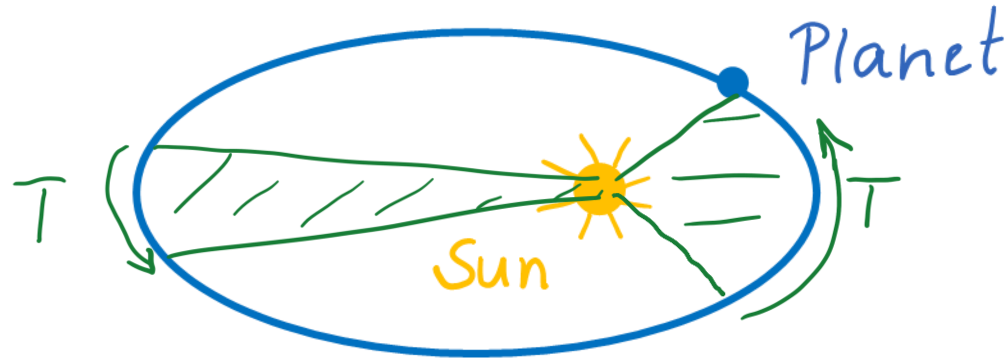
Example 3

A merry-go-round (solid disk of mass M and radius R) is rotating on frictionless bearings about a vertical axis through its center. It rotates in the clockwise direction with angular speed ω. A child of mass $\frac{1}{2}M$ is initially sitting at the outer edge of the merry-go-round. When the child jumps off tangentially to the circumference, the merry-go-round reverses its rotation and now rotates with the same angular speed ω in the opposite, i.e. counterclockwise, direction. Derive an expression for the speed relative to the ground with which the child jumps off.



Kepler's 2nd Law

A line drawn between the sun and a planet sweeps out equal areas in equal intervals of time



Torque by gravity on the planet is zero.
Angular momentum is conserved.

$$\frac{dA}{dt} = \frac{l}{2m} = constant$$

(Derived in lecture)