Lecture 25: Problem solving review
For test 3

Concepts: Rotation

- Relationship angular- linear quantities
- Rolling without slipping
- Moment of Inertia, parallel axis theorem
- Rotational kinetic energy
- Torque
- Angular dynamics
- Static equilibrium
- Angular momentum conservation

How to identify type of problem?

If object is not moving at all, or moving at constant velocity: no acceleration, no angular acceleration, Static Equilibrium

If external forces and torques act on object(s):

- acceleration and angular acceleration can be obtained from sum of forces/sum of torques
- speed can be obtained from Energy/Work
- speed can be obtained from acceleration+kinematics only if forces/torques are constant

If no external torques act (e.g. rotational collisions):

Angular momentum conserved, mechanical energy changes

Energy problems

Identify motion of each object:

Only translating → $K = K_{trans} = ½Mv^2$

Only rotating → $K = K_{rot} = ½\, I\, \omega^2$

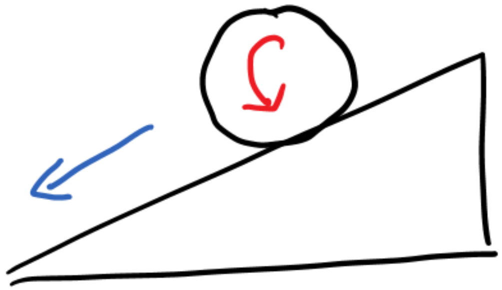
Both rotating and translating → $K = K_{trans} + K_{rot}$

$K = ½Mv^2 + ½\, I\, \omega^2$

No slipping: $v = \omega R$

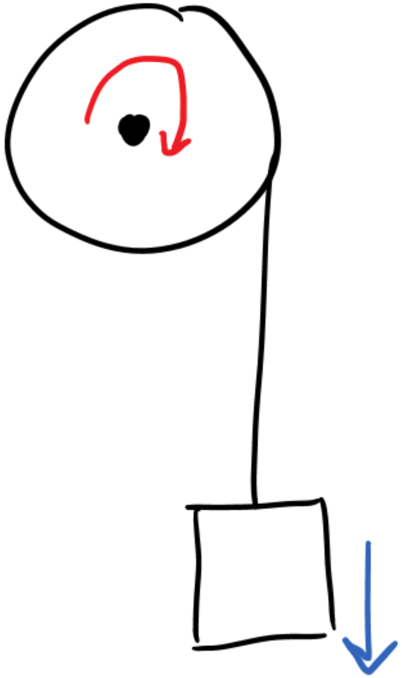
Identify other energies

Ball rolling down incline



Translation + Rotation + Potential energy of gravity

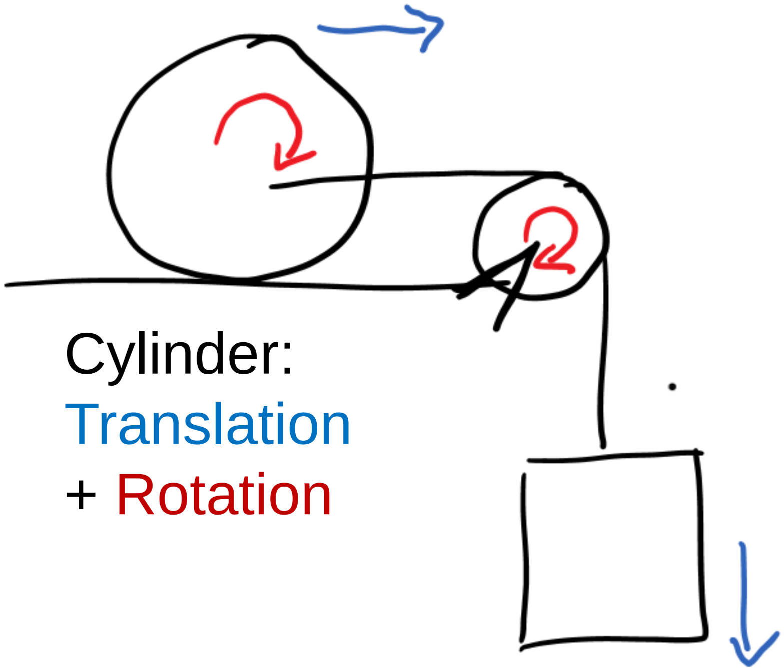
Box suspended from fixed pulley; string unwinds, box descends



Pulley: Rotation

Box: Translation

+ Potential energy of gravity



Cylinder:
Translation
+ Rotation

Pulley: Rotation

Box: Translation

+ Potential energy of gravity for box

Example 1

You have a pumpkin of mass *M* and radius *R*. The pumpkin has the **shape of a sphere**, but it is **not uniform** inside; so you do not know its moment of inertia.

In order to determine the moment of inertia, you decide to roll the pumpkin down an incline that makes an angle θ with the horizontal. The pumpkin starts from rest and rolls without slipping. When it has descended a vertical height *H*, it has acquired a speed $V = \sqrt{\frac{5}{4}gH}$.

Use energy methods to derive an expression for the moment of inertia of the pumpkin.

Forces and torques

Draw extended free-body diagram

Identify motion of object:

Object can rotate → $\sum \tau_z = I\alpha_z$

Object can translate → $\Sigma\vec{F} = m\vec{a}$

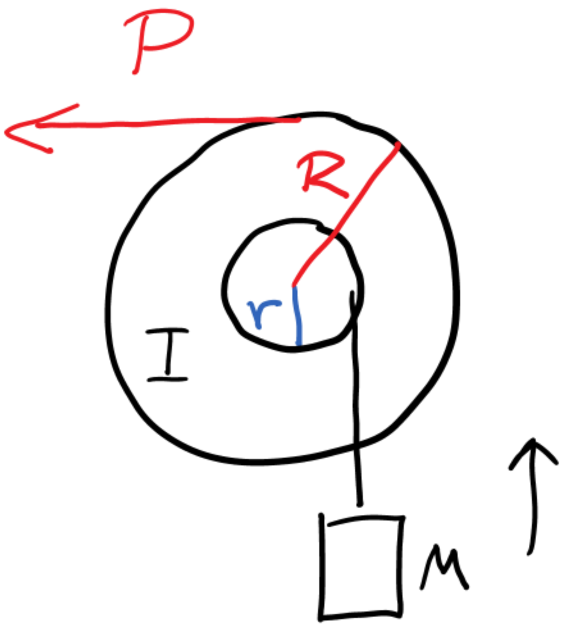
Object can do both → $\sum \tau_z = I\alpha_z$ and $\Sigma\vec{F} = m\vec{a}$

No slipping: $a = \alpha R$

Example 2

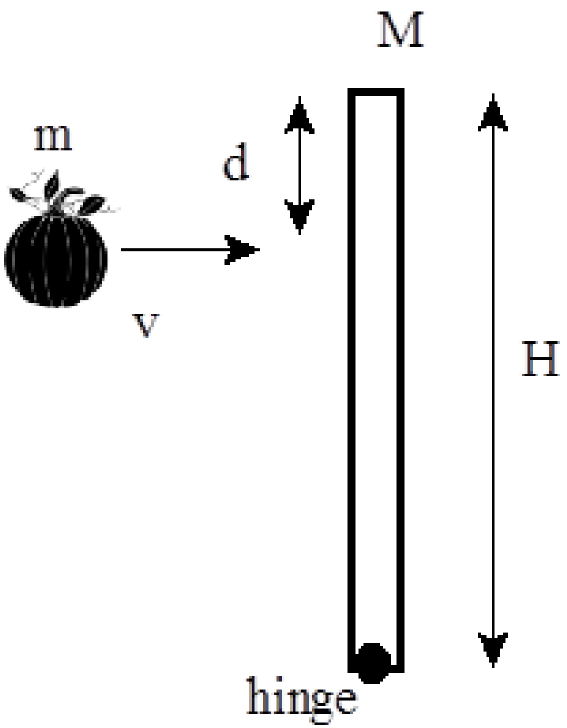
A yo-yo shaped device (moment of inertia about center is *I*) is mounted on a horizontal frictionless axle through its center is used to lift a load of mass *M*. The outer radius of the device is *R*, the radius of the hub is *r*. A constant horizontal force of magnitude *P* is applied to a rope wrapped around the outside of the device. The box, which is suspended from a rope wrapped around the hub, accelerates upwards. The ropes do not slip.

Derive an expression for the acceleration of the box.



Example 3

In a pumpkin throwing contest, a small pumpkin of mass *m* is moving horizontally with speed *v* when it hits a vertical pole of length *H* and mass *M* that is pivoted at a hinge at its foot. The pumpkin hits the pole a distance *d* from its upper end and becomes impaled on a long nail that is sticking out of the pole. The pumpkin is small enough to be treated as a point mass.



Derive an expression for the angular speed of the system after the collision.

A statics example

A box of weight ½*W* hangs from the top end of a uniform post that is pivoted on the ground at an angle θ with respect to the horizontal. A horizontal rope is tied to the post a quarter of the way from the top end. The length of the post is *L* and its weight is *W*. The tension in the horizontal rope is 2*W*.

Derive an expression for angle θ in terms of system parameters. Simplify your answer.