Lecture 27: Wave motion

- longitudinal and transverse waves
- traveling waves
- wave length, frequency, and speed
- distinction between wave speed and speed of a particle
- speed of a wave on a string
- Doppler effect

The Physlet simulations presented in this lecture have been developed by, and are used with permission from, Davidson College.

http://webphysics.davidson.edu/Applets/Applets.html

What is a wave?

A wave is a self-propagating disturbance in a medium.

The material of the medium is **not** moved along the wave. Particles of the medium are momentarily displaced from their equilibrium.

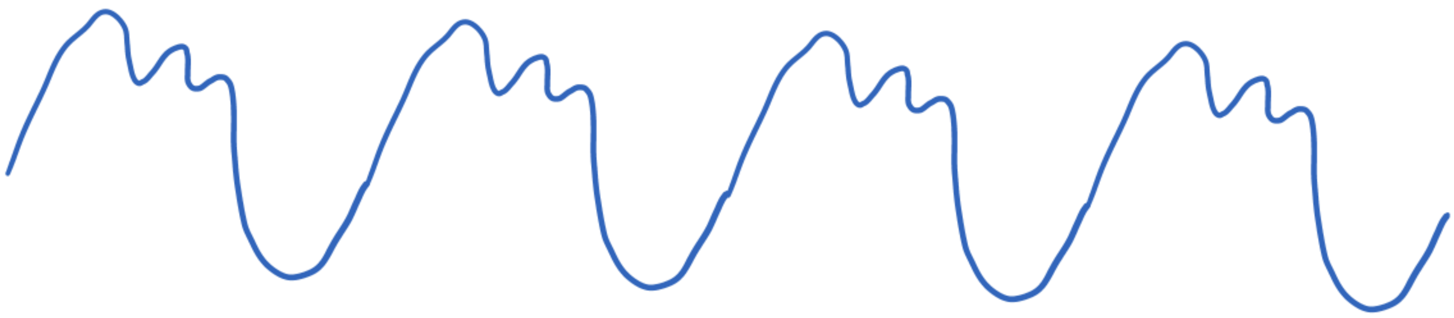
Transverse and longitudinal waves

Transverse wave:
displacement **perpendicular** to propagation direction
(ex: wave on string, water wave)

Longitudinal wave:
displacement **parallel** to propagation direction
(ex: sound)

Simulation

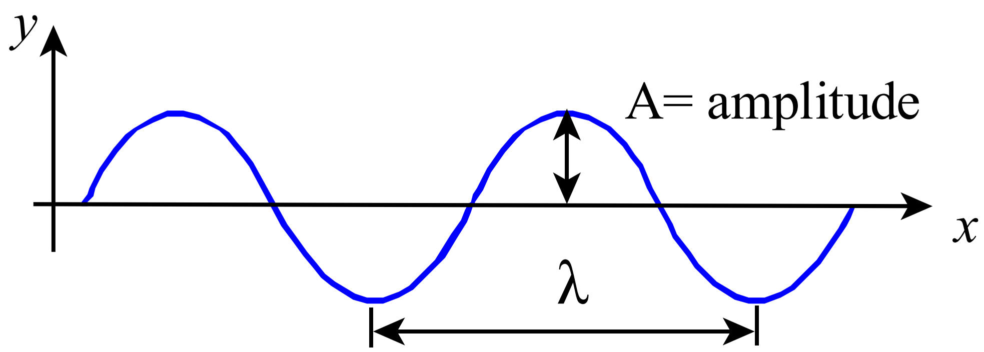
Periodic waves



We will only study sine-shaped waves, because all periodic functions can be expressed as a superposition of different sine functions.

Sinusoidal waves

Snapshot at fixed time:

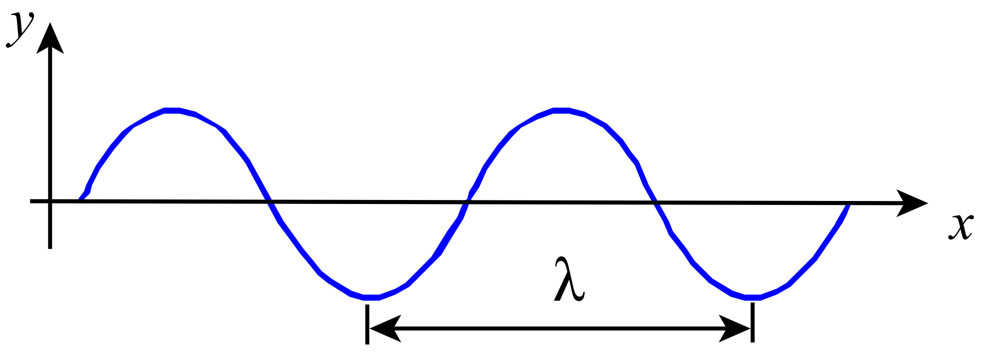


wavelength

$$y(x) = A\sin(kx + \varphi)$$

k wave number

Wave length and wave number



$$y(x+\lambda) = y(x)$$

$$A\sin(k(x+\lambda)+\varphi) = A\sin(kx+\varphi)$$

$$k(x+\lambda)+\varphi = kx+\varphi+2\pi$$

$$k\lambda = 2\pi$$

$$k = \frac{2\pi}{\lambda}$$

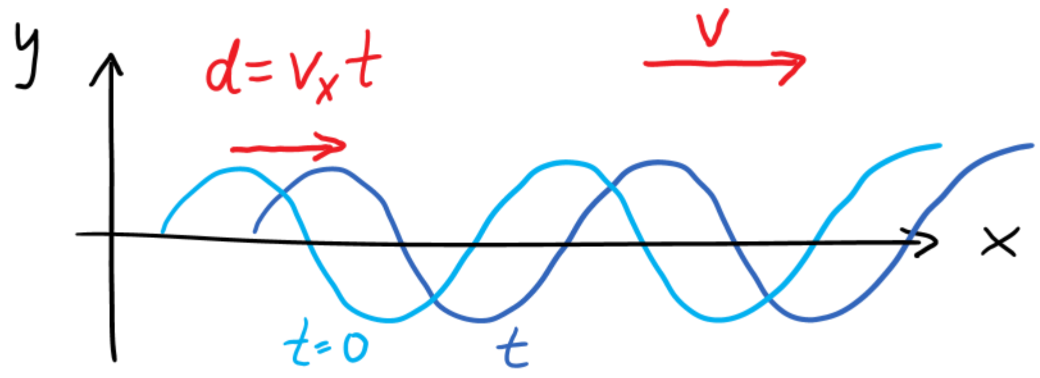
Time dependence at fixed position

Particle located at x=0:

$$y(t) = A\sin(\omega t + \varphi)$$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

Traveling wave



At fixed t: $y(x) = A\sin(kx + \varphi)$

Wave traveling with v_x:

$y(x,t) = A\sin(k(x - v_x t) + \varphi)$

$y(x,t) = A\sin(kx \pm \omega t + \varphi)$

$$\omega = kv = 2\pi f = \frac{2\pi}{T}$$

Use

$-\omega$ for $v_x = +v$ moving in positive x-dir.

$+\omega$ for $v_x = -v$ moving in negative x-dir.

Wave speed and direction

$$y(x,t) = A\sin(kx \pm \omega t + \varphi)$$

Use

$-\omega$ for $v_x = +v$ (moving in positive x-direction)

$+\omega$ for $v_x = -v$ (moving in negative x-direction)

$$v = \frac{\omega}{k} = \lambda f = \frac{\lambda}{T}$$

$$k = \frac{2\pi}{\lambda}$$

$$\omega = 2\pi f$$

The wave travels one wavelength during one period.

$$y(x,t) = A\sin(kx \pm \omega t + \varphi)$$

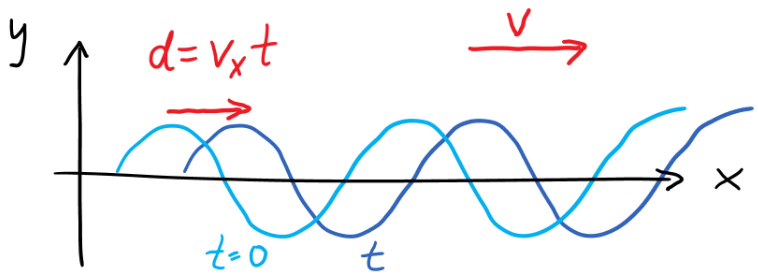
Use

$-\omega$ for $v_x = +v$ (moving in positive x-direction)

$+\omega$ for $v_x = -v$ (moving in negative x-direction)*

*because $v_x = -\frac{\partial y}{\partial t} / \frac{\partial y}{\partial x}$

Crest: $y(x,t) = A = constant, \frac{dv}{dy} = 0$



$$\frac{dv}{dy} = \frac{\partial y}{\partial x}\frac{dx}{dt} + \frac{\partial y}{\partial t} = 0$$

$$v_x = \frac{dx}{dt} = -\frac{\frac{\partial y}{\partial t}}{\frac{\partial y}{\partial x}} = -\frac{\pm\omega}{k}$$

Transverse velocity

Velocity of a particle

y-displacement: $y(x,t) = A\sin(kx \pm \omega t + \varphi)$

Caution: Particle performs harmonic motion along the $\boldsymbol{y}$-direction, not along the x-direction of the wave motion $\Longrightarrow$ need v_y

For particle at fixed position x:

$$v_y = \frac{\partial y}{\partial t} = \pm A\omega\cos(kx \pm \omega t + \varphi) \neq v_x$$

Maximum transverse speed

$$v_y = \frac{\partial y}{\partial t} = \pm A\omega \cos(kx \pm \omega t + \varphi)$$

$$v_{ymax} = A\omega$$

Example

If $y(x,t) = 3\sin(2x + 8t + \frac{1}{4}\pi)$ (in SI units),

1. What is the speed and direction of this traveling wave?

Wave is moving at 4 m/s in the negative-x direction.

2. What is the maximum speed of a particle in the medium?

Maximum speed of the particle is 24 m/s.

Speed of transverse wave on string

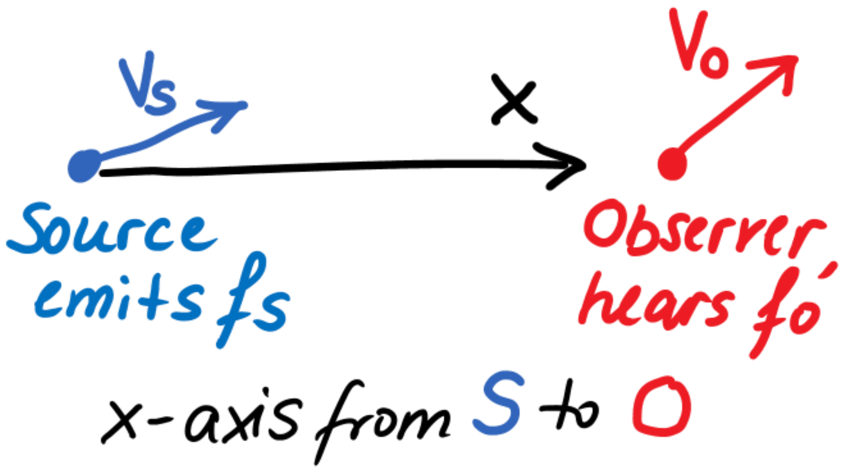
$$v = \frac{\omega}{k} = \sqrt{\frac{F_T}{\mu}}$$

F_T Tension in the string

μ linear mass density mass per unit length

Caution: speed of a transverse wave ≠ transverse speed

Doppler Effect



$$f'_o = f_S \frac{v - v_{ox}}{v - v_{sx}}$$

v = speed of the waves in the medium

Example for Doppler Effect

You are exploring a planet in a very fast ground vehicle. The speed of sound in the planet's atmosphere is 250 m/s. You are driving straight toward a cliff wall at 50 m/s. In panic, you blow your emergency horn to warn the cliff wall to get out of your way.

If the frequency of your horn is 1000 Hz, what is the frequency your helpless friend who is standing by the cliff wall will hear?

What is the frequency of the sound you hear reflected off the cliff before you crash into it?

Beats

If two waves traveling in the same direction have frequencies f_1 and f_2 very close to one another, they will produce beats in amplitude with a frequency equal to their frequency difference:

$$f_{Beat} = |f_1 - f_2|$$

The combined sound will "warble" with the beat frequency.

Demo: 440Hz and 441 Hz