Lecture 28: Wave interference

- Superposition of waves
- Standing waves on a string
- Interference

Standing Waves

Two waves traveling in opposite directions with:

- same amplitude
- same wavelength λ (and thus same k)
- same frequency f (and thus same ω)

$\Longrightarrow$ same speed $v = \lambda f$.

$y_1(x,t) = A_0 \sin(kx - \omega t)$ traveling in the positive x-dir.

$y_2(x,t) = A_0 \sin(kx + \omega t)$ traveling in the negative x-dir.

Equation for Standing Waves

$$y_1(x,t) = A_0 \sin(kx - \omega t)$$ traveling in the positive x-dir.

$$y_2(x,t) = A_0 \sin(kx + \omega t)$$ traveling in the negative x-dir.

$$\sin a + \sin b = 2 \sin \frac{a+b}{2} \cos \frac{a-b}{2}$$

$$y_1 + y_2 = 2\,A_0 \sin \frac{(kx - \omega t) + (kx + \omega t)}{2} \cos \frac{(kx - \omega t) - (kx + \omega t)}{2}$$

$$= 2\,A_0 \sin \frac{(2kx)}{2} \cos \frac{(-2\omega t)}{2}$$

$$\boxed{y(x,t) = y_1 + y_2 = 2A_0 \sin(kx)\, cos(\omega t)}$$ Standing wave

Standing wave on a string

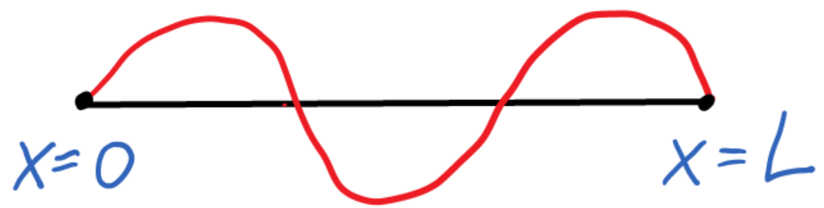
$$y(x,t) = A\sin(kx)\,cos(\omega t)$$

String of length L with fixed ends:

$y(x=0,t)=0$

$y(x=L,t)=0$

Boundary Condition



$\Longrightarrow \sin(kL)=0 \qquad kL = n\pi$

With $k=\frac{2\pi}{\lambda}$: $\boxed{\lambda = \frac{2L}{n}}$

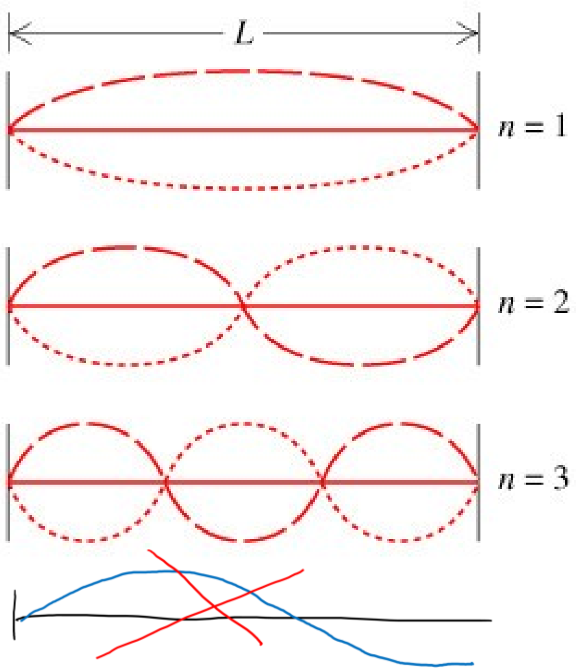
and with $v = f\lambda$: $\boxed{f = \frac{n\,v}{2L}}$

Fundamental frequency and harmonics

String of length L
with fixed ends

$$\lambda = \frac{2L}{n}$$

$$f = \frac{n\,v}{2L}$$



n=1: fundamental, first harmonic:
λ =2L, $f = \frac{v}{2L}$

n=2: 2nd harmonic, 1st overtone:
λ =L, $f = \frac{v}{L}$

n=3: 3rd harmonic, 2nd overtone:
$\lambda = \frac{2}{3}$L, $f = \frac{3v}{2L}$

Example 1

A wire has a length of 8m. The speed of waves on the wire is 240m/s. What is the fundamental frequency?

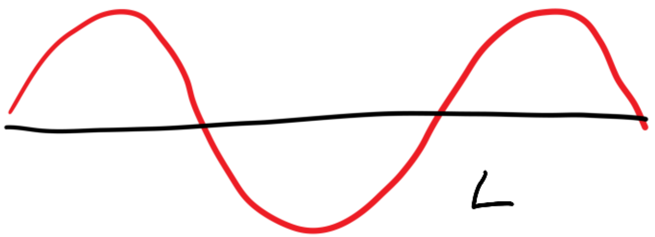
Example 2

A particular guitar string has a mass of 3.0 grams and a length of 0.75 m. A standing wave on the string has the shape shown in the figure. The wave has a frequency of 1200 Hz.

(a) What is the speed of the wave?

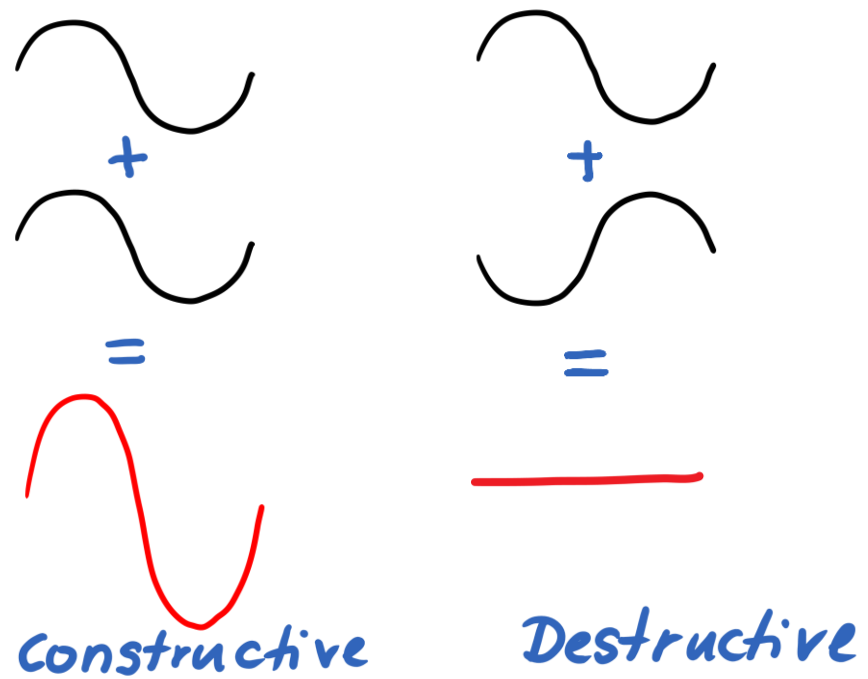
(b) What is the tension of the string?

(c) The wave on the string produces a sound wave. Does the sound wave have the same frequency, wavelength, or speed as the wave on the string?

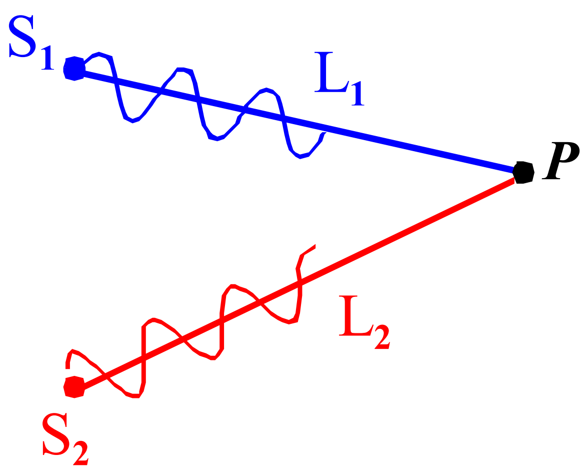


Interference

Two or more traveling waves superimpose: interference



Interference and path length



Two sources, waves emitted in phase. How do waves combine at P?

If crests of both waves arrive at same time:
constructive interference
integer n

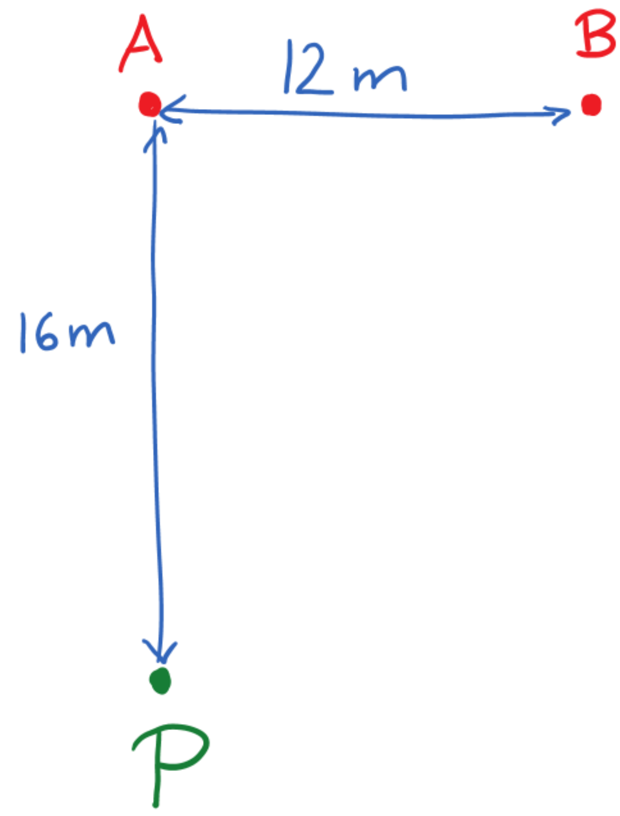
$$|L_1 - L_2| = \Delta L = n\lambda$$

If crest of wave 1 arrives at same time as trough of wave 2:
destructive interference

$$|L_1 - L_2| = \Delta L = (n + ½)\,\lambda$$

Example 3

Two radio transmitters (**A** and **B**) are 12 m apart. They are driven by the same oscillator (i.e., they emit in phase) and generate waves of wavelength 2 m. How do the waves interfere at point **P** that is 16 m directly in front of source A?



Example 4

Two loudspeakers emit waves of frequency 172 Hz. The speed of sound is 344 m/s. You are 8 m from speaker A. How close can you get to speaker B and have destructive interference?