Lecture 30: 2nd Law of Thermodynamics

- Thermodynamic cycles
- 2nd law of Thermodynamics
- Carnot Cycle

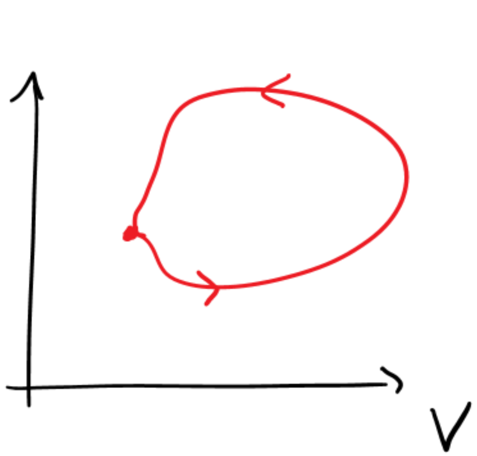
Thermodynamic Cycles

System returns to initial state

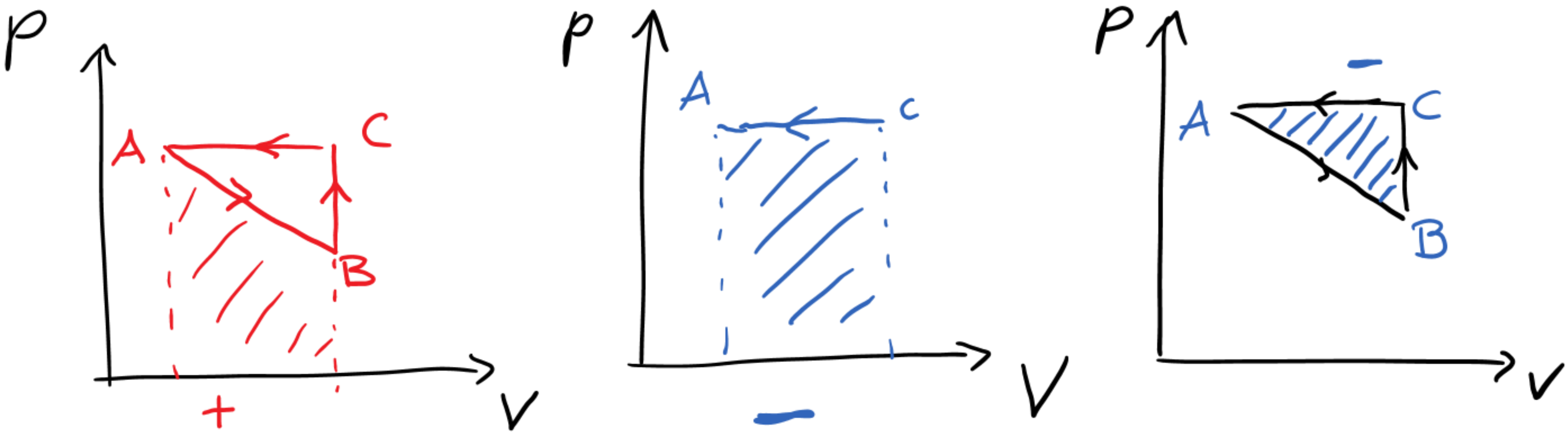
$$U_f = U_i$$

$$\Delta U = Q - W = 0$$

$$\boxed{Q = W}$$



Work in cycles



W= area enclosed in the cycle

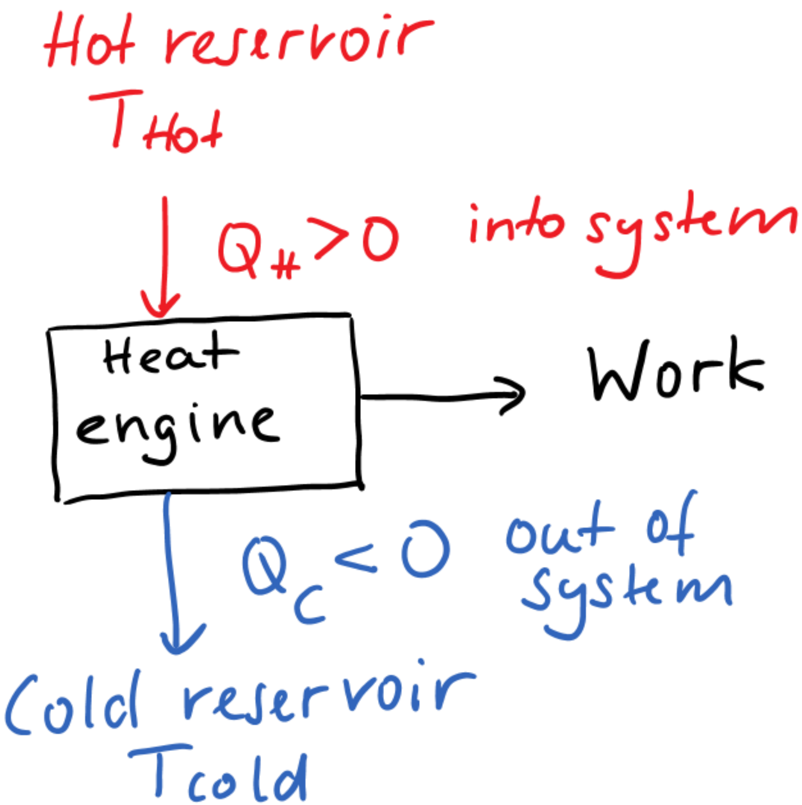
Clockwise: more positive W than negative W

$$W_{net} > 0$$

Counter-clockwise: more negative W than positive W

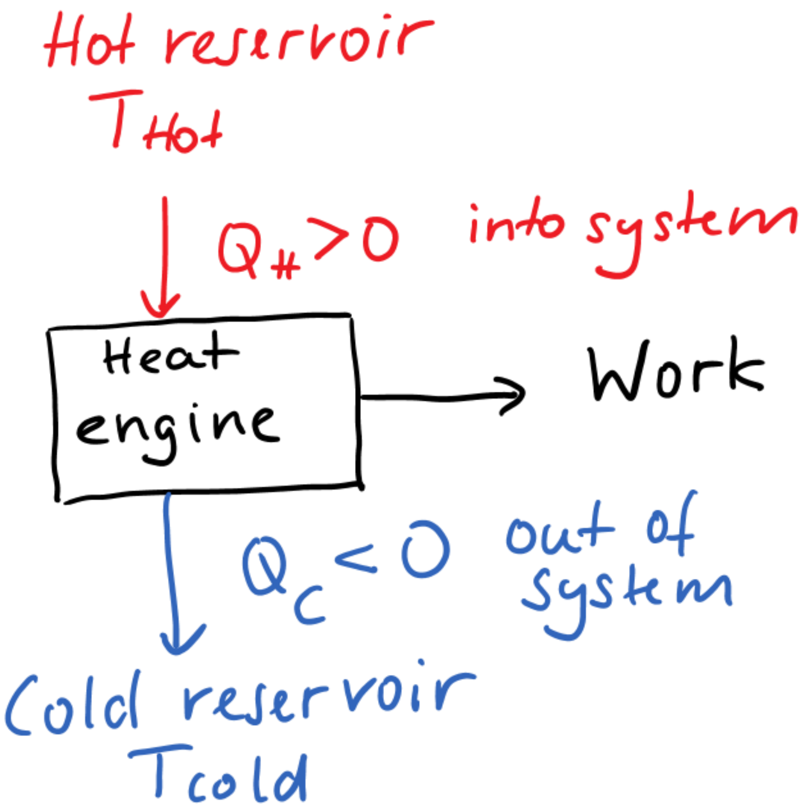
$$W_{net} < 0$$

Heat engine



$$Q_{net} = Q_H + Q_C$$
$$= Q_H - |Q_C|$$

Efficiency



$$e = \frac{W_{out}}{Q_H} = \frac{Q_{net}}{Q_H} = \frac{Q_H + Q_C}{Q_H}$$

$$e = 1 - \frac{|Q_C|}{Q_H}$$

2^{nd} Law of Thermodynamics

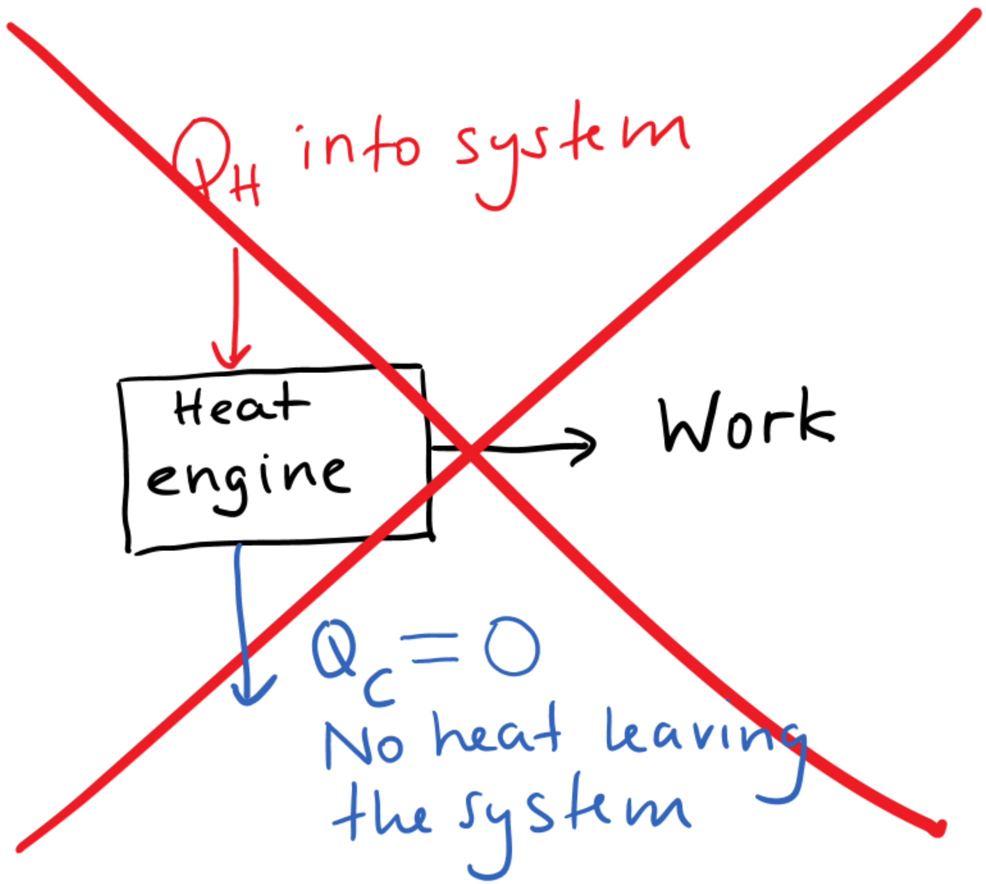
Clausius:

Heat flows naturally from a hot object to a cold object; heat will not flow spontaneously from a cold object to a hot object.

Kelvin-Planck:

No device is possible whose sole effect is to transform a given amount of heat completely into work.

impossible to construct perpetual motion machine of 2^{nd} kind

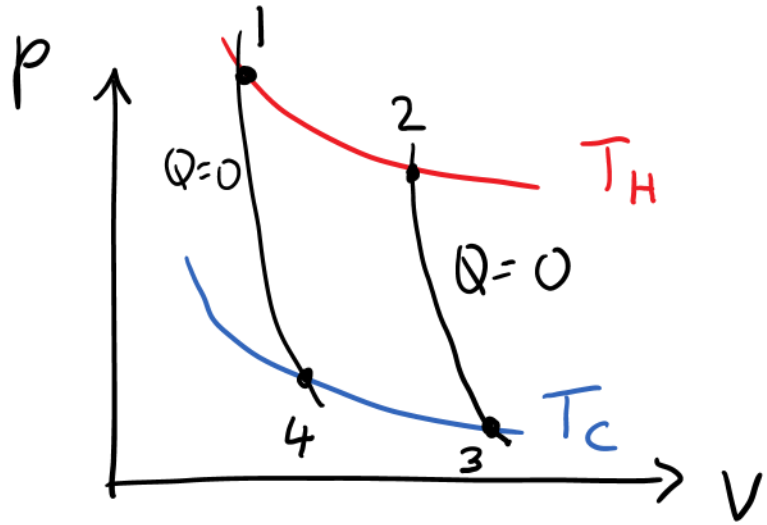


Forbidden by 2nd Law

$$e = 1 - \frac{|Q_C|}{Q_H}$$

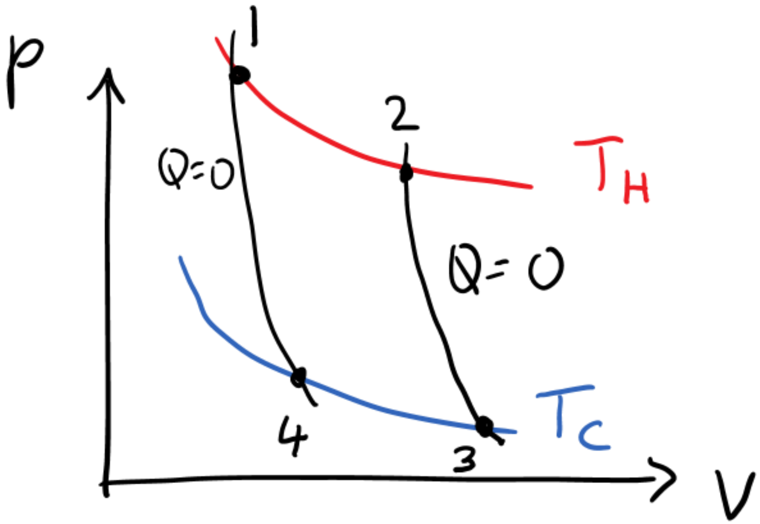
$e = 1$ is impossible

Carnot Cycle



1-2 isothermal expansion
2-3 adiabatic expansion
3-4 isothermal compression
4-1 adiabatic compression

Heat and Work in Carnot Cycle



1-2 isothermal expansion

$$\Delta U = 0$$

$$Q = W = \int p\, dV = \int_1^2 \frac{nRT}{V} dV$$

$$Q = W = nRT_H \ln \frac{V_2}{V_1} > 0$$

2-3 adiabatic expansion

$$Q = 0$$

$$\Delta U = -W$$

$$W = -nc_V(T_C - T_H)$$

3-4 isothermal compression

$$\Delta U = 0,\ Q = W = nRT_C \ln \frac{V_4}{V_3} < 0$$

4-1 adiabatic compression

$$Q = 0, \Delta U = -W,\ W = -nc_V(T_H - T_C)$$

Efficiency of Carnot Cycle

1-2 isothermal expansion: $Q = W = nRT_H \, ln\frac{V_2}{V_1} > 0$

2−3 adiabatic expansion: $Q = 0, W = -nc_V(T_C - T_H)$

3-4 isothermal compression: $Q = W = nRT_C \, ln\frac{V_4}{V_3} < 0$

4−1 adiabatic compression: $Q = 0, W = -nc_V(T_H - T_C)$

$$e = \frac{W_{out}}{Q_H} = \frac{nRT_H \, ln\frac{V_2}{V_1} + nRT_C \, ln\frac{V_4}{V_3}}{nRT_H \, ln\frac{V_2}{V_1}} = 1 + \frac{T_C \, ln\frac{V_4}{V_3}}{T_H \, ln\frac{V_2}{V_1}} = 1 - \frac{T_C \, ln\frac{V_3}{V_4}}{T_H \, ln\frac{V_2}{V_1}}$$

With adiabatic equation and some math:

$$e_{Carnot} = 1 - \frac{T_C}{T_H}$$

Carnot Cycle has maximum efficiency

Heat transfer during isothermal process reversible
No heat transfer during process that involves temperature change

Carnot cycle is reversible

If more efficient engine existed:
Couple hypothetical engine with reverse Carnot engine
Transforms amount of heat completely into work
Violates 2nd Law

More efficient engine can not exist

$$e_{Carnot} = 1 - \frac{T_C}{T_H}$$

Maximum efficiency of any cycle operating between T_C and T_H