

Rec Sec Number _____

TEST 1 (4 pages)

and First Name: _____

For questions on this page, write the letter which you believe to be the best answer in the underlined space provided **to the left of the question number**.

For problems on subsequent pages: your solution to a question with *OSE* in front of it must begin with an *Official Starting Equation*. The expression for the

final result must be in system parameters and simplified as far as possible. Draw a box around your answer to each question. Neglect air resistance. Calculators and notes cannot be used during the test. If you have any questions, ask the proctor. **You must put your name on each page of the test.**

Test Total = 200 / 200

B 1. (10 points) The y -position of a particle is given by the function $y(t) = p \sin(qt) + rt^3$, where p , q and r are positive constants. What can be said about the particle's velocity and acceleration in the y direction at time $t = 0$?

A) $v_y = 0$, $a_y > 0$ B) $v_y > 0$, $a_y = 0$

C) $v_y = 0$, $a_y = 0$ D) $v_y > 0$, $a_y < 0$

$$v_y = pq \cos qt + 3rt^2 > 0 @ t=0$$

$$a_y = -pq^2 \sin qt + 6rt = 0 @ t=0$$

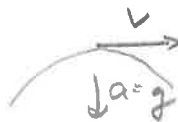
B 2. (10 points) A child kicks a ball from the ground with an initial velocity directed at an angle > 0 above the horizontal. When the ball reaches its maximum height, which is true?

A) Its acceleration is zero.

B) Its acceleration is perpendicular to its velocity.

C) Its speed is zero.

D) Its speed has the same value as the initial speed.



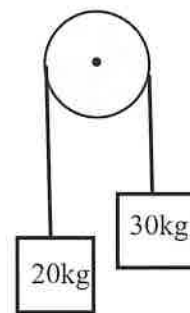
A 3. (10 points) A 20kg box and a 30kg crate are attached to the two ends of a massless string that passes over a massless frictionless pulley. The system is released from rest. Which of the following is **true** about the acceleration of the system?

A) $a < g$

B) $a > g$

C) $a = g$

D) $a = 0$



B 4. (10 points) A box is in the middle of a flat-bed truck. The truck is speeding up on a straight, level road, and the box remains in the middle of the truck bed. The force of friction on the box is _____ and aimed in the _____ direction as the truck's motion.

A) static, opposite

B) static, same

C) kinetic, opposite

D) kinetic, same

B 5. (10 points) A particle moves in a circle with a constant radius R , and its initial centripetal acceleration is a . If the speed of the particle is tripled while keeping the radius the same, what will be the new centripetal acceleration?

A) $3a$

B) $9a$

C) $\frac{1}{3}a$

D) $\frac{1}{9}a$

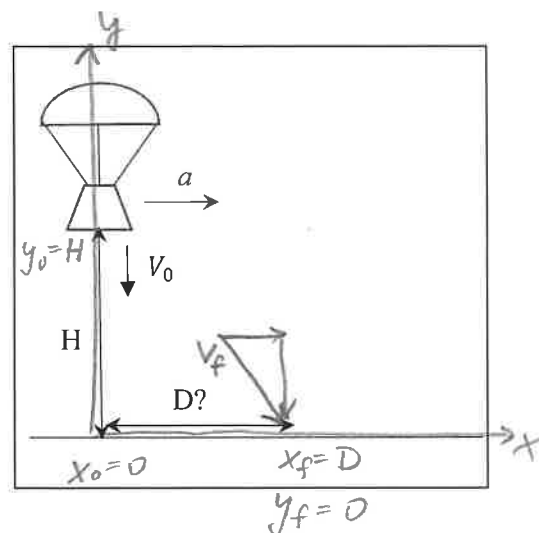
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Name: Solution

6. A landing capsule is descending over the Atlantic Ocean. When it is at height H above the water, it has a velocity V_0 that is directed vertically down. Because of the parachutes, the capsule experiences **no vertical acceleration**. However, the wind causes the capsule to have a constant horizontal acceleration a directed to the right.

a) (10 points) Complete the diagram with all the required information to solve parts b and c.

(OSE) b) (25 points) In terms of system parameters, derive an expression for the distance D the capsule will have traveled horizontally by the time it touches the water.



Time to water: $y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$
 $0 = H - v_0 t$
 $t = \frac{H}{v_0}$

Horizontal distance in this time:

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$$

$$D = \frac{1}{2}a t^2$$

$$D = \frac{1}{2}a \left(\frac{H}{v_0} \right)^2$$

(OSE) c) (15 points) In terms of system parameters, derive an expression for the velocity just before the capsule touches the water, **in unit vector notation**. You may use the distance D as a system parameter for this part.

$$v_x = v_{x0} + a_x t$$

$$v_x = \frac{aH}{v_0}$$

$$v_y = v_{y0} + a_y t$$

$$v_y = -v_0$$

OR

$$v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$$

$$v_y = -v_0$$

OR

$$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$$

$$v_x = \sqrt{2aD}$$

$$\vec{v} = \frac{aH}{v_0} \hat{i} - v_0 \hat{j}$$

$$\vec{v} = \sqrt{2aD} \hat{i} - v_0 \hat{j}$$

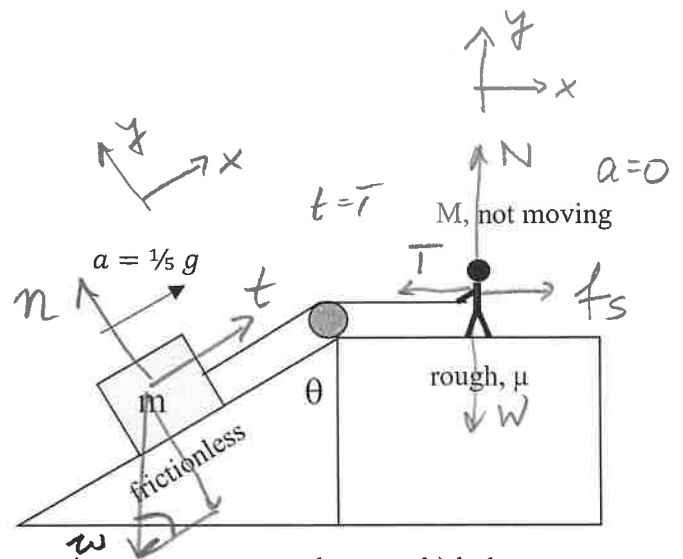
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Name: Solution

7. (50 points) A student of mass M is standing on a **rough** horizontal surface that has a coefficient of static friction μ with the student's shoes. The student is pulling a box of mass m up a ramp by means of a massless rope that is parallel to the surface and runs over a massless, frictionless pulley. The **frictionless ramp** makes an angle θ with the vertical. The box is speeding up with an acceleration of magnitude $\frac{1}{5}g$.



a) (10 points) In the figure, superimpose complete free-body diagrams for the student and the box, including all information necessary to solve part b) below.

(OSE) b) (40 points) Derive an expression, in terms of system parameters, for the maximum mass m of the box for which the student can remain at rest.

$$M: \sum F_x = T_x + N_x + f_{sx} + W_x = M a_x$$

$$-T + f_s = 0 \Rightarrow T = f_s$$

$$\sum F_y = T_y + N_y + f_{sy} + W_y = M a_y$$

$$N - Mg = 0 \Rightarrow N = Mg$$

$$m: \sum F_x = N_x + t_x + w_x = m a_x$$

$$T - mg \cos \theta = m \left(\frac{1}{5} g \right)$$

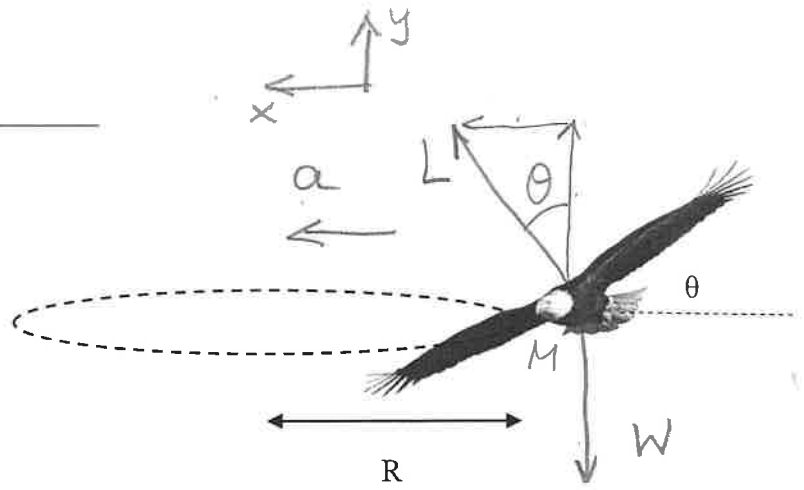
Maximum mass m : static friction maxed out
 $f_s = \mu_s N$

$$\Rightarrow \mu_s Mg - mg \cos \theta = \frac{1}{5} mg$$

$$m = \frac{\mu_s M}{\cos \theta + \frac{1}{5}}$$

Name: Solution

8. (50 points) Birds experience a lift force that is **perpendicular to the wings**. An eagle is flying in a **horizontal** circle of radius R at constant speed. The eagle achieves this by tilting his wings at angle θ with respect to the horizontal.



a) (10 points) Draw a fully labeled free-body diagram for the eagle, including all information necessary to solve part b).

(OSE) b) (40 points) Derive an expression for the speed of the eagle in terms of system parameters.

$$\sum F_x = L_x + \cancel{W_x} = Ma_x$$

$$L \sin \theta = M \frac{v^2}{R}$$

$$\sum F_y = L_y + W_y = \cancel{Ma_y^0}$$

$$L \cos \theta - Mg = 0$$

$$L \cos \theta = Mg$$

$$\tan \theta = \frac{v^2}{Rg}$$

$$v = \sqrt{Rg \tan \theta}$$