Fall 2024 **Physics 1135** Print LAST Name: Solution

Rec Sec Number _____ **TEST 2 (4 pages)** and First Name: ______________

For questions on this page, write the letter which you believe to be the best answer in the underlined space provided **to the left of the question number**. For problems on subsequent pages: your solution to a question with *OSE* in front of it must begin with an *O*fficial *S*tarting *E*quation. The expression for the final result must be in system parameters and simplified as far as possible. Draw a box around your answer to each question. Neglect air resistance. Calculators and notes cannot be used during the test. If you have any questions, ask the proctor. **You must put your name on each page.**

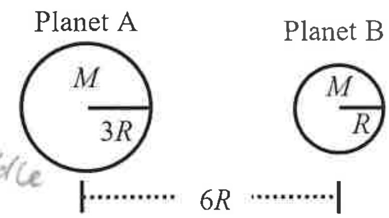
Test Total = 200 **/ 200**

C 1. (10 points) A particle enters a region with a potential energy given by $U(x) = ax^3 - bx$ where x is the position and a and b are positive constants. What is the x-component of the force at $x = 0$?

A) a B) $-a$ C) b D) $-b$

$F_x = -\frac{dU}{dx} = -(3ax^2 - b)$ $x = 0$: $F_x = b$

A 2. (10 points) A spaceship sits on a line which connects the centers of planet A (mass M, radius $3R$) and planet B (mass M, radius R). The line is a length of $6R$. What is the distance from the center of planet A such that the net force on the spaceship is zero?



A) $3R$ B) R

C) $5R$ D) $4R$

Same mass $\rightarrow$ forces balance in the middle

A 3. (10 pts) A tennis ball of mass m and a football of mass $2m$ have the same linear momentum. Which one has the larger kinetic energy?

A) The tennis ball. B) The football. C) They have the same kinetic energy.

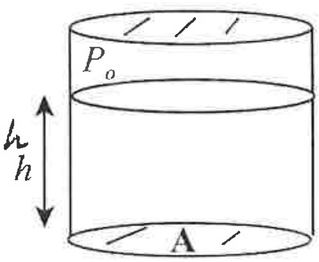
D) Can't say because the information given is not sufficient.

$mv_1 = 2mv_2$

$v_1 = 2v_2$

$\frac{1}{2}mv_1^2 = \frac{1}{2}m(2v_2)^2 > \frac{1}{2}(2m)v_2^2$

A 4. (10 points) A closed tank contains a fluid of unknown density. The depth of the fluid is h and the area of the bottom is A. The tank is pressurized, so that the pressure at the top of the tank is P_O. The pressure at the bottom of the tank is $4P_O$. What is the density of the fluid?



$\Delta p = 3p_0 = \rho g h$

A) $3P_o/(gh)$ B) $4P_o/(gh)$ C) $3gh/P_o$ D) $3P_o A$

B 5. (10 points) An object of weight W has a density four times the density of water. It is hung from a vertical spring force scale and lowered into a vat of water. When the object is fully submerged and not moving, the reading on the scale is:

A) ⅓W B) ¾ W C) 3 W D) W

B ↑↑ F_s ↓ W

$F_s + B = W$

$F_s = W - B$

$= 4\rho g V - \rho g V$

$= 3\rho g V = \frac{3}{4} W$

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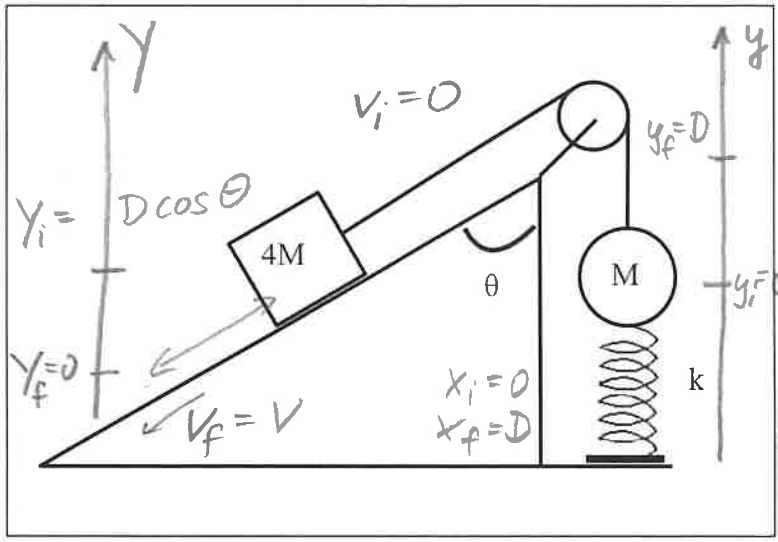
6. A block of mass $4M$ is on a frictionless incline that makes some **unknown** angle θ with respect to the vertical as shown in the figure. It is connected to a ball of mass M by a massless string that passes over a massless, frictionless pulley. The ball is attached to a massless spring of spring constant k; the other end of the spring is secured to the ground. The system is released from rest when the spring is in equilibrium position. Afterwards, the block descends while the ball rises, stretching the spring.



a) (10 points) Complete the diagram with all information necessary to solve part b below.

b) (40 points) (OSE) When the block has travelled a distance D along the incline, it has speed V. Using energy methods, derive a symbolic expression, in terms of relevant system parameters, for the angle θ the incline makes with the vertical.

$$E_f - E_i = \cancel{W_{other}} \to 0$$

$$E_i = E_f$$

$$\frac{1}{2}4M\cancel{v_i^2} + \frac{1}{2}M\cancel{v_i^2} + 4MgY_i + Mg\cancel{y_i} + \frac{1}{2}k\cancel{x_i^2} =$$

$$\frac{1}{2}4Mv_f^2 + \frac{1}{2}Mv_f^2 + 4Mg\cancel{Y_f} + Mgy_f + \frac{1}{2}kx_f^2$$

$$4MgD\cos\theta = \frac{5}{2}MV^2 + MgD + \frac{1}{2}kD^2$$

$$\boxed{\cos\theta = \frac{\frac{5}{2}MV^2 + MgD + \frac{1}{2}kD^2}{4MgD}}$$

$$\text{or:} \quad \theta = \arccos\left[\frac{\frac{5}{2}MV^2 + MgD + \frac{1}{2}kD^2}{4MgD}\right]$$

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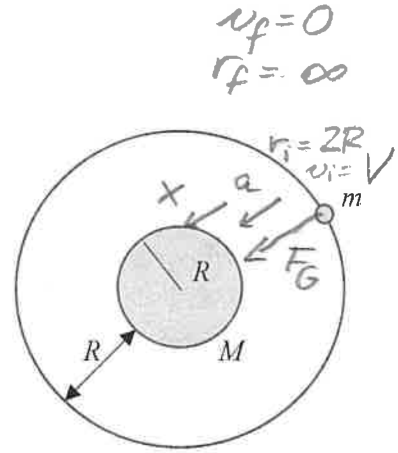
7. A space-probe of mass m is initially orbiting a planet of mass M, radius R at a distance R above the planet's surface.



a) (10 points) Draw the free body diagram for the satellite and include all the required information to solve parts b and c below.

b) (20 points) (OSE) Derive a symbolic expression for the speed V of the probe, in terms of system parameters.

$$\sum F_x = m a_x$$

$$\frac{GMm}{(2R)^2} = m\frac{v^2}{2R}$$

$$\frac{GM}{2R} = v^2$$

$$\boxed{v = \sqrt{\frac{GM}{2R}}}$$

c) (20 points) (OSE) To escape the planet's gravity, the engines of the probe are fired producing W_{eng} amount of work. Find the minimum value of W_{eng}. Assume mass of the probe is unchanged in the process. **Use V as a system parameter for this part.**

$$E_f - E_i = W_{eng}$$

$$\cancel{\tfrac{1}{2}mv_f^2}^{\,0} - \cancel{\frac{GMm}{r_f}}^{\,0} - \left(\tfrac{1}{2}mv_i^2 - \frac{GMm}{r_i}\right) = W_{eng}$$

$$W_{eng} = -\tfrac{1}{2}mV^2 + \frac{GMm}{2R}$$

or

$$W_{eng} = \frac{GMm}{2R} - \tfrac{1}{2}m\left(\frac{GM}{2R}\right) = \frac{GMm}{4R}$$

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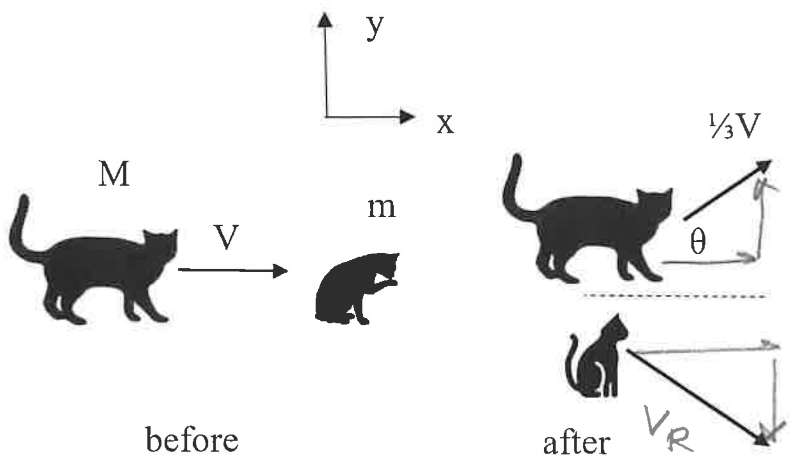
8. Dr. Vojta's kittens are playing on a horizontal floor. Raisin of mass m is originally sitting still, licking her paws. Frodo of mass M is moving with speed V in the positive x-direction and collides with Raisin. Immediately after the collision, Frodo is skidding with speed $\frac{1}{3}V$ at angle θ above the positive x-axis, while Raisin is sliding with an unknown velocity.



a) (40 points) (OSE) In terms of system parameters, derive an expression for Raisin's velocity **immediately** after the collision, **in unit vector notation.**

$$\cancel{J_{net\,ext}}^{\,0} = \vec{P}_f - \vec{P}_i$$

$$P_{ix} = P_{fx}$$

$$MV = M\tfrac{1}{3}V\cos\theta + m v_{Rx}$$

$$v_{Rx} = \frac{MV(1-\frac{1}{3}\cos\theta)}{m}$$

$$P_{iy} = P_{fy}$$

$$0 = M\tfrac{1}{3}V\sin\theta + m v_{Ry}$$

$$v_{Ry} = -\frac{\frac{1}{3}MV\sin\theta}{m}$$

$$\boxed{\vec{V}_R = \frac{MV}{m}(1-\tfrac{1}{3}\cos\theta)\,\hat{\imath} - \tfrac{1}{3}\frac{M}{m}V\sin\theta\,\hat{\jmath}}$$

$$\text{or}\quad \vec{V}_R = \frac{MV}{m}\left\{(1-\tfrac{1}{3}\cos\theta)\,\hat{\imath} - \tfrac{1}{3}\sin\theta\,\hat{\jmath}\right\}$$

b) (10 points) (OSE) Derive an expression for the x-component of the impulse delivered **to** Frodo **by** Raisin.

$$J_x = p_{fF_x} - p_{iF_x} = M\tfrac{1}{3}V\cos\theta - MV$$

$$\boxed{J_x = MV(\tfrac{1}{3}\cos\theta - 1)}$$

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