

For questions on this page, write the letter which you believe to be the best answer in the underlined space provided **to the left of the question number**. For problems on subsequent pages: your solution to a question with *OSE* in front of it must begin with an *Official Starting Equation*. The expression for the final result must be in system parameters and simplified as far as possible. Draw a box around your answer to each question. Neglect air resistance. Calculators and notes cannot be used during the test. If you have any questions, ask the proctor. **You must put your name on each page.**

Test Total = 200 / 200

C 1. (10 points) A uniform solid cylinder, a uniform solid sphere, and hoop with the same mass and radius are placed at the top of an incline and released from rest. They are rolling without slipping. Which one is the first to reach the bottom of the incline?

- A) The hoop B) the cylinder C) the solid sphere D) all three at the same time

C 2. (10 points) A rotating dinner turntable has a sugar cube in the center, a saltshaker halfway out from the center, and pepper mill near the outer edge. Which of the following is true?

- A) The pepper and the sugar have the same linear velocity as the salt.
 B) The pepper has a larger angular velocity than the salt.
 C) The pepper has a larger linear velocity than the salt.
 D) The sugar has a larger angular velocity than the salt.

C 3. (10 points) If the amplitude of a simple harmonic oscillator is tripled, its period is

- A) reduced by a factor of 3^2 B) tripled C) unchanged D) increased six-fold

A 4. (10 points) A uniform rod of mass M and length L is pivoted at a point half-way between its end and its center of mass. Its period for small oscillations is

A $2\pi \sqrt{\frac{7L}{12g}}$

B) $2\pi \sqrt{\frac{19L}{48g}}$

C) $2\pi \sqrt{\frac{7L}{48g}}$

D) $2\pi \sqrt{\frac{L}{g}}$

$$T = 2\pi \sqrt{\frac{\frac{7}{48} ML^2}{Mg(\frac{L}{4})}}$$

$$I = I_{CM} + m\left(\frac{L}{4}\right)^2 = \frac{1}{12} ML^2 + \frac{1}{16} ML^2 = \frac{7}{48} ML^2$$

$$T = 2\pi \sqrt{\frac{7L}{12g}}$$

$$T = 2\pi \sqrt{\frac{I}{MgD}}$$

C 5. (10 points) A block on a spring is undergoing simple harmonic motion with amplitude A . When it is one-fourth of the way to the maximum distance from its equilibrium position, what fraction of its total mechanical energy is potential energy?

A) $\frac{1}{4}$

B) $\frac{3}{4}$

C) $\frac{1}{16}$

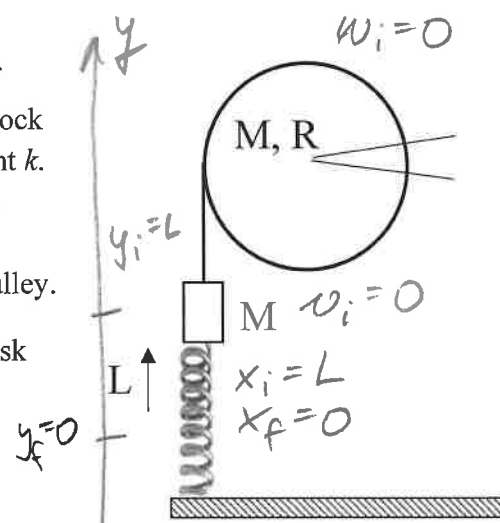
D) $\frac{2}{3}$

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Name: Solution

6. (50 points) A string is wound tightly around a disk-shaped pulley of mass M and radius R . The free end of the string is then attached to a block of mass M , which itself is attached to a vertical spring of spring constant k . The disk is rotated clockwise by some external agent until the spring is stretched from its equilibrium length by a distance L . It is held in this position until it is released from rest. The string does not slip on the pulley.

Use energy methods to derive an expression for angular speed of the disk when the spring is again at its equilibrium length.



$$E_f - E_i = W_{\text{other}}^0$$

$$E_i = E_f$$

$$\frac{1}{2} M v_i^2 + \frac{1}{2} I \omega_i^2 + M g y_i + \frac{1}{2} k x_i^2 = \frac{1}{2} M v_f^2 + \frac{1}{2} I \omega_f^2 + M g y_f + \frac{1}{2} k x_f^2$$

$$M g L + \frac{1}{2} k L^2 = \frac{1}{2} M v_f^2 + \frac{1}{2} \left(\frac{1}{2} M R^2 \right) \omega_f^2$$

No slipping: $v = \omega R$

$$M g L + \frac{1}{2} k L^2 = \underbrace{\frac{1}{2} M \omega_f^2 R^2 + \frac{1}{4} M R^2 \omega_f^2}_{\frac{3}{4} M R^2 \omega_f^2}$$

$$\omega_f = \sqrt{\frac{M g L + \frac{1}{2} k L^2}{\frac{3}{4} M R^2}} = \sqrt{\frac{4(M g L + \frac{1}{2} k L^2)}{3 M R^2}}$$

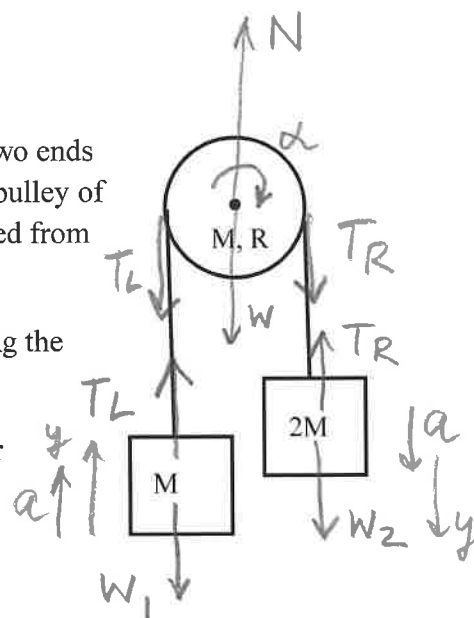
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7. (50 points) Two boxes of masses M and $2M$, respectively, are attached to the two ends of a massless string as shown in the figure. The string passes over a disk-shaped pulley of mass M and radius R . The string does not slip on the pulley. The system is released from rest.

a) (10 points) Draw the free-body diagrams for each of the three objects, including the acceleration and coordinate systems needed to solve part b below.

b) (OSE) (40 points) Use forces and torques to derive an expression for the linear acceleration of the boxes, in terms of system parameters.



$$M: \sum F_y = T_{Ly} + W_{1y} = Ma_y$$

$$T_L - Mg = Ma$$

$$2M: \sum F_y = T_{Ry} + W_{2y} = 2Ma_y$$

$$-T_R + 2Mg = 2Ma$$

$$\text{Pulley: } \sum \tau_z = \tau_{T_L z} + \tau_{T_R z} + \cancel{\tau_{N z}} + \cancel{\tau_{W z}} = I \alpha_z$$

$$-T_L R + T_R R = \frac{1}{2} MR^2 \alpha$$

$$\text{No slip: } a = \alpha R$$

$$\text{Combine: } -T_L R + T_R R = \frac{1}{2} MR^2 \frac{a}{R}$$

$$-T_L + T_R = \frac{1}{2} Ma$$

$$-(Ma + Mg) + 2Mg - 2Ma = \frac{1}{2} Ma$$

$$Mg = Ma\left(\frac{1}{2} + 2 + 1\right)$$

$$T_L = Ma + Mg$$

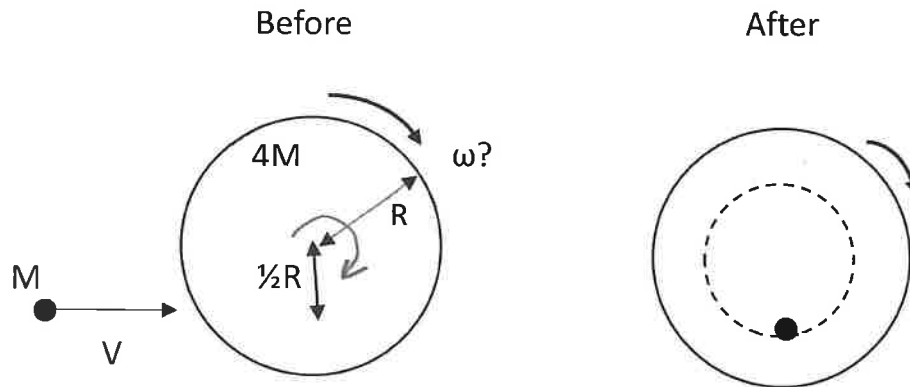
$$T_R = 2Mg - 2Ma$$

$$a = \frac{2}{7} g$$

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8. (50 pts) A disk-shaped merry-go-round of mass $4M$ and radius R is rotating clockwise with some unknown angular speed about a fixed frictionless axis passing through its center. A child of mass M jumps on to the merry-go-round with speed v as illustrated and lands at a point half-way between the center and the edge. Afterwards, the merry-go-round rotates with **half** its initial angular speed in the clockwise direction.



a) (10 points) Calculate the moment of inertia of the merry-go-round with the child on it.

$$I = I_{\text{disk}} + I_{\text{child}}$$

$$I = \frac{1}{2} 4M R^2 + M \left(\frac{1}{2}R\right)^2 = \frac{4}{2} MR^2 + \frac{1}{4} MR^2$$

$$I = \frac{9}{4} MR^2$$

b) (40 points) Derive an expression for the disk's initial angular speed before the collision, in terms of system parameters.

$$\sum \tau_{\text{ext},z} = 0 \Rightarrow L_{i,z} = L_{f,z}$$

$$L_{\text{disk},z} + L_{\text{child},z} = L_{\text{tot},z}$$

$$\frac{1}{2} 4MR^2 \omega - Mv \frac{1}{2} R = \frac{9}{4} MR^2 \frac{\omega}{2}$$

$$2MR^2 \omega - \frac{9}{8} MR^2 \omega = \frac{1}{2} MvR$$

$$\left(\frac{16}{8} - \frac{9}{8}\right) MR^2 \omega = \frac{1}{2} MvR$$

$$\frac{7}{8} R \omega = \frac{1}{2} v$$

$$\boxed{\omega = \frac{4}{7} \frac{v}{R}}$$