

Rec Sec Number \_\_\_\_\_

**TEST 1 (4 pages)**

and First Name: \_\_\_\_\_

For questions on this page, write the letter which you believe to be the best answer in the underlined space provided **to the left of the question number**.

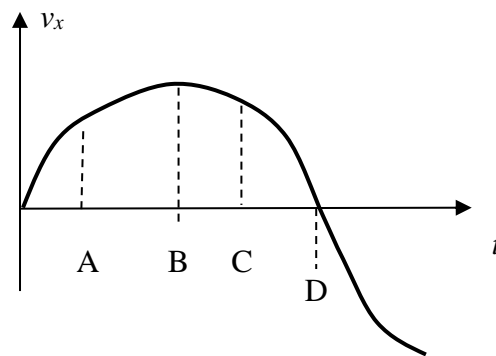
For problems on subsequent pages: your solution to a question with *OSE* in front of it must begin with an *Official Starting Equation*. The expression for the final result must be in system parameters and simplified as far as possible.

Draw a box around your answer to each question. Neglect air resistance. Calculators and notes cannot be used during the test. If you have any questions, ask the proctor. **You must put your name on each page of the test.**

Test Total = <u>200</u> / 200
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D 1. (10 points) The  $v_x$ - $t$  graph for an object that is moving along the  $x$ -axis is shown on the right. Which of the following is true?

- A) The object is slowing down at A.
- B) The object is changing direction at B.
- C) The object is speeding up at C.
- D) The object reaches its maximum  $x$ -position at D.



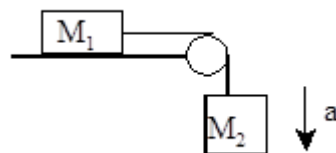
B 2. (10 points) An object is moving along the  $y$ -axis. The  $y$ -component of the object's velocity as a function of time  $t$  is given by

$v_y(t) = bt^2 - c$  where  $b$  and  $c$  are positive constants. What can be said about the object's velocity and acceleration at  $t = 0$ ?

- A)  $v_y = 0, a_y = 0$
- B)  $v_y < 0, a_y = 0$
- C)  $v_y < 0, a_y > 0$
- D)  $v_y > 0, a_y = 0$

A 3. (10 points) A cart of mass  $M_1$  on a horizontal frictionless table is connected to a block of mass  $M_2$  by a rope that runs over a massless, frictionless pulley. The block accelerates downward. We know that the tension  $T$  in the rope:

- A)  $T < M_2 g$
- B)  $T = M_2 g$
- C)  $T > M_2 g$
- D)  $T = M_1 g$



C 4. (10 points) A block is at rest on a rough inclined plane. The reaction force to the weight force on the block is ...

- A) the vertical component of the normal force.
- B) the force of static friction.
- C) the force of gravity exerted by the block on the Earth.
- D) the force exerted by the block on the incline.

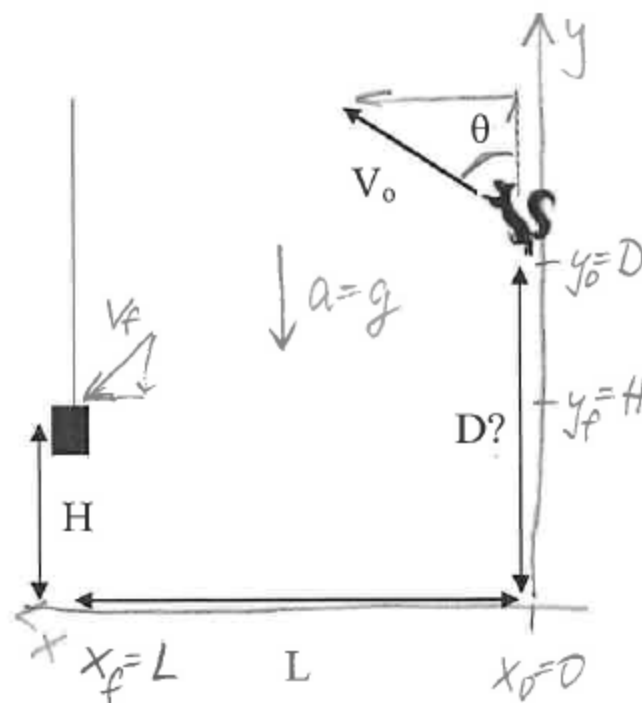
A 5. (10 points) A small toy car is driving vertical circles inside a hollow cylinder at constant speed. At what position does the normal force acting on the car have the smallest magnitude?

- A) at the top
- B) at the bottom
- C) when the car is exactly halfway to the top
- D) the normal force is the same everywhere.

<u>50</u> / 50 for this page
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6. A squirrel wants to reach a bird feeder that hangs at height  $H$  above the ground, a horizontal distance  $L$  away from a tree. The squirrel climbs up the tree to some unknown height and jumps with initial velocity  $V_0$  directed at an angle  $\theta$  with respect to the vertical, as shown in the figure. The squirrel's launch point is higher than the feeder.



a) (10 points) Complete the diagram on the right with all information necessary to solve the parts below.

b) (25 points) (OSE) Derive an expression for the vertical height  $D$  above the ground from which the squirrel needs to launch to arrive at the feeder, in terms of system parameters.

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$

$$L = v_0 \sin \theta t$$

$$t = \frac{L}{v_0 \sin \theta}$$

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

$$H = D + v_0 \cos \theta t + \frac{1}{2}(-g)t^2$$

$$H = D + v_0 \cos \theta \left( \frac{L}{v_0 \sin \theta} \right) - \frac{1}{2}g \left( \frac{L}{v_0 \sin \theta} \right)^2$$

$$D = H - L \cot \theta + \frac{1}{2}g \left( \frac{L}{v_0 \sin \theta} \right)^2$$

c) (15 points) (OSE) Derive an expression, in unit vector notation, for the velocity with which the squirrel arrives at the feeder, in terms of system parameters.

$$v_x = v_{0x} + a_x t \quad v_y = v_{0y} + a_y t$$

$$\text{or: } v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

$$v_y = -\sqrt{(v_0 \cos \theta)^2 - 2g(H - D)}$$

$$v_x = v_0 \sin \theta \quad v_y = v_0 \cos \theta - g \left( \frac{L}{v_0 \sin \theta} \right)$$

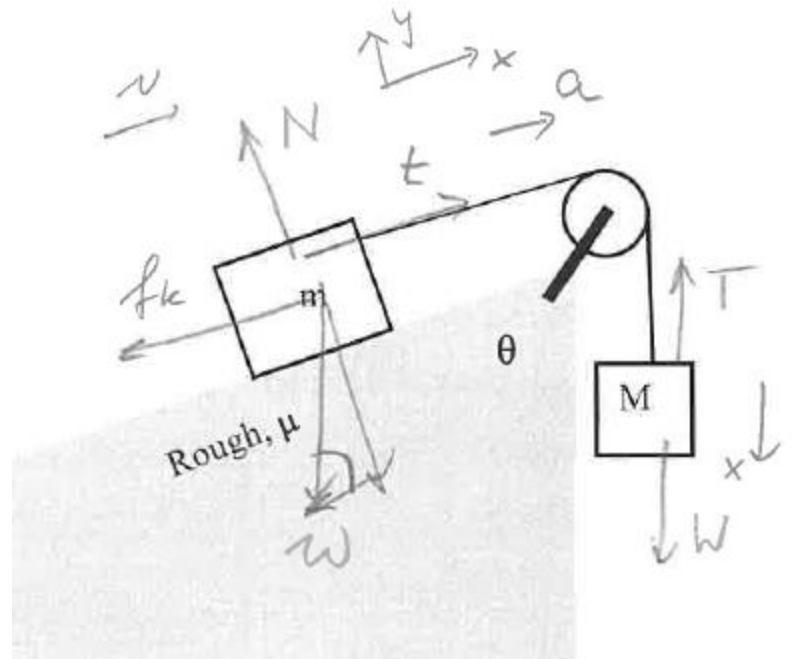
with  $D$  from part b

$$\vec{v} = v_0 \sin \theta \hat{i} + \left( v_0 \cos \theta - \frac{gL}{v_0 \sin \theta} \right) \hat{j}$$

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7. (50 points) Two blocks are connected by a massless string. One block of mass  $m$  is moving up a rough inclined plane that makes angle  $\theta$  with the vertical. The block and the plane have a coefficient of kinetic friction  $\mu$  between them. The other block of mass  $M$  hangs over a massless frictionless pulley.

a) (10 points) In the figure, superimpose fully labeled free-body diagram for both blocks. Include all information necessary to solve part b) below.



b) (OSE) (40 points) Derive an expression for the tension in the string, in terms of system parameters.

$$M: \sum F_x = T_x + W_x = \text{Max}$$

$$-T + Mg = Ma_x$$

$$m: \sum F_x = t_x + N_x + w_x + f_{kx} = ma_x$$

$$t - mg \cos \theta - f_k = ma_x \quad (t = T)$$

$$f_k = \mu N$$

$$\sum F_y = t_y + N_y + w_y + f_{ky} = ma_y$$

$$N - mg \sin \theta = 0$$

$$N = mg \sin \theta$$

$$\rightarrow a_x = -\frac{T}{M} + g$$

$$\rightarrow T - mg \cos \theta - \mu mg \sin \theta = m \left( g - \frac{T}{M} \right)$$

$$T \left( 1 + \frac{m}{M} \right) = mg + mg \cos \theta + \mu mg \sin \theta$$

$$T = \frac{mg(1 + \cos \theta + \mu \sin \theta)}{1 + \frac{m}{M}}$$

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8. (50 points) A block of mass  $M$  is attached to a vertical rod by means of two strings of equal length  $L$ . When the system rotates about the axis of the rod, the strings are extended as shown in the diagram, and the tension in the upper string is  $T_u$ . Both strings are taut and at an angle  $\theta$  with respect to the rod.

Note: the tension in the lower string is *unknown*.

a) (10 points) Draw a complete free-body diagram for the block including all information that you need to solve the task below. Any symbol you use in your calculations must appear in the diagram.

b) (OSE) (40 points) In terms of the relevant system parameters, derive an expression for the speed of the block.

$$\Sigma F_x = W_x + T_{Lx} + T_{ux} = Max$$

$$T_u \sin \theta + T_L \sin \theta = M \frac{v^2}{R}$$

$$\Sigma F_y = W_y + T_{uy} + T_{Ly} = May$$

$$-Mg + T_u \cos \theta - T_L \cos \theta = 0$$

$$T_L = T_u - \frac{Mg}{\cos \theta}$$

$$\rightarrow T_u \sin \theta + \left( T_u - \frac{Mg}{\cos \theta} \right) \sin \theta = M \frac{v^2}{R}$$

$$2T_u \sin \theta - Mg \tan \theta = \frac{Mv^2}{R} \text{ with } R = L \sin \theta$$

$$V = \sqrt{\frac{L \sin \theta}{M} [2T_u \sin \theta - Mg \tan \theta]}$$

