

Rec Sec Number \_\_\_\_\_

**TEST 2 (4 pages)**

and First Name: \_\_\_\_\_

For questions 1-5, write your answer in the underlined space **to the left of the question number**. For problems 6-8: begin with an Official Starting Equation. The final expression must be in system parameters and simplified as far as possible. Neglect air resistance. Calculators and notes cannot be used during the test. If you have questions, ask the proctor. **You must put your name on each page.**

Test Total = <u>200</u> / 200
-------------------------------

B 1. (10 points) A large balloon is made of an effectively massless high-tech material. It is then filled with helium of volume  $V$ . It floats in the air but is attached to the ground with a massless string. The density of air is  $\rho_{\text{air}}$ , and the density of helium is  $\rho_{\text{He}}$ . What is the tension in the string?

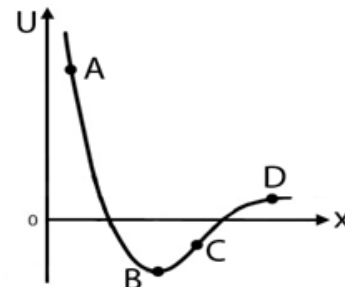
- A)  $\rho_{\text{He}} V g$       B)  $(\rho_{\text{air}} - \rho_{\text{He}}) V g$       C)  $(\rho_{\text{He}} - \rho_{\text{air}}) V g$       D)  $\rho_{\text{air}} V g$

C 2. (10 points) A cylinder of height  $H$  and base area  $A$  with uniform density is floating upright in water of density  $\rho$ . 25% of the cylinder remains **above** the waterline. The difference between the pressure at the cylinder's lower surface and the atmospheric pressure at the water surface equals

- A)  $\frac{1}{4} \rho g H$       B)  $\rho g H$       C)  $\frac{3}{4} \rho g H$       D) 0

C 3. (10 points) A particle is moving along the  $x$ -axis under the influence of a conservative force whose potential energy is shown in the figure. At which point is the largest force directed to the left?

- A) A      B) B      C) C      D) D



C 4. (10 points) The mass of a certain planet is eight times the mass of the earth. On this planet the free fall acceleration is  $2g$ . In terms of the radius of the earth,  $R_E$ , the radius of this planet is:

- A)  $8R_E$       B)  $4R_E$       C)  $2R_E$       D)  $\frac{1}{2} R_E$

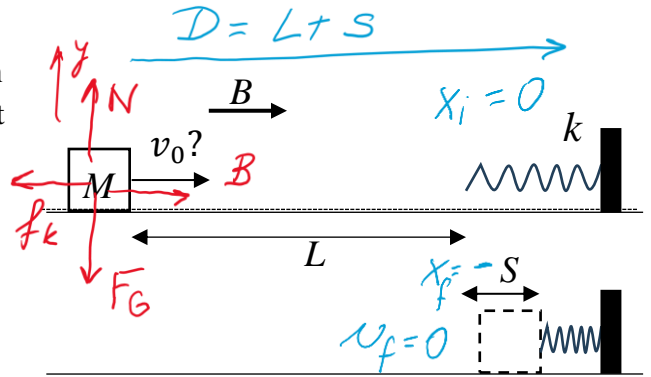
D 5. (10 points) A tennis ball moving to the right collides with a stationary billiard ball. After the collision, the tennis ball moves to the left. The billiard ball moves very slowly to the right. Which of the following statements is true about the collision?

- A) The tennis ball experiences a larger impulse than the billiard ball.  
 B) The billiard ball experiences a smaller force than the tennis ball.  
 C) The tennis ball experiences a smaller impulse than the billiard ball.  
 D) Both experience the same magnitude of impulse.

_____ / 50 for this page
--------------------------

Name: \_\_\_\_\_

6. (50 points) A block of mass  $M$  is launched at some unknown initial speed on a rough horizontal surface that has a coefficient of kinetic friction  $\mu$ . After traveling a distance  $L$  it comes in contact with a spring of spring constant  $k$  that is attached to a wall. The block hits the spring and compresses it a distance  $S$  before it momentarily comes to rest. Throughout the trip, a constant horizontal force  $B$  acts on the block as shown.



a) (10 points) Complete the diagram with all information necessary to solve part b below.

b) (OSE) (40 points) Derive an expression for the initial speed of the block, in terms of system parameters.

$$E_f - E_i = W_{\text{other}}$$

$$\frac{1}{2} M V_f^2 + \frac{1}{2} k x_f^2 - \frac{1}{2} M V_0^2 - \frac{1}{2} k x_i^2 = W_B + W_f$$

$$\frac{1}{2} k S^2 - \frac{1}{2} M V_0^2 = \vec{B} \cdot \vec{D} + \vec{f} \cdot \vec{D}$$

$$= B(L+S) - \mu N(L+S)$$

$$\text{Find } N: \Sigma F_y = M a_y = 0$$

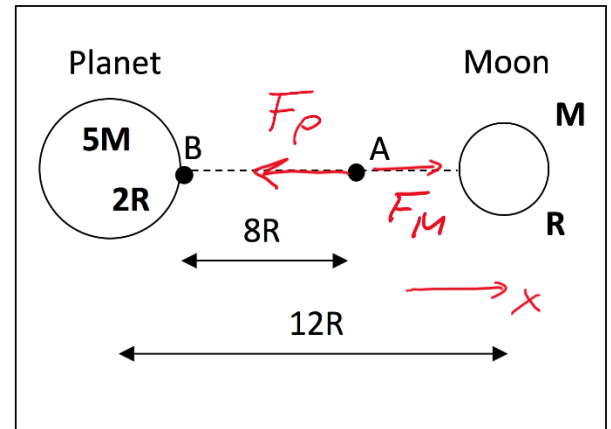
$$N = Mg$$

$$\frac{1}{2} M V_0^2 = \frac{1}{2} k S^2 + B(L+S) - \mu Mg(L+S)$$

$$V_0 = \sqrt{\frac{2}{M} \left[ \frac{1}{2} k S^2 + (\mu Mg - B)(L+S) \right]}$$

Name: \_\_\_\_\_

7. (50 points) A rocket of mass  $m$  and speed  $V$  is at point A which is a distance  $8R$  from the surface of a planet of mass  $5M$  and radius  $2R$ . The rocket is directly on a line between the planet and its moon which has mass  $M$  and radius  $R$ . The center-to-center distance between the planet and the moon is  $12R$ . Ignore the orbital motion of moon and planet.



a) (15 points) In terms of relevant system parameters, derive an expression for the net force on the rocket when it is at point A with its engines shut off.

$$\Sigma F_x = F_{px} + F_{Mx}$$

$$F_{net_x} = -\frac{G \cdot 5Mm}{(10R)^2} + \frac{GMm}{(2R)^2}$$

$$F_{net_x} = \frac{GMm}{R^2} \left( \frac{-5}{100} + \frac{1}{4} \right) = \frac{GMm}{R^2} \left( \frac{-1+5}{20} \right) = \frac{GMm}{5R^2}$$

b) (35 points) Now the engines start, and the rocket travels towards the planet. In terms of relevant system parameters, derive an expression for the work done by the engines if the rocket travels from point A to the planet and lands with zero speed at point B (see figure).

$$E_f - E_i = W_{other} = W_{eng}$$

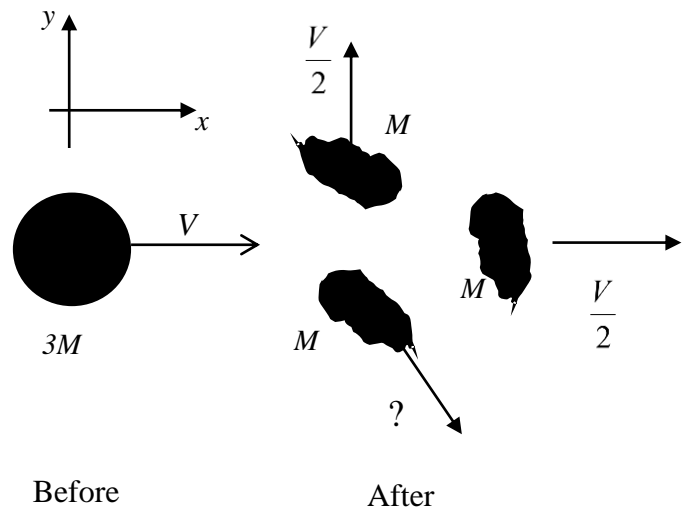
$$\frac{1}{2}mv_f^2 - \frac{G5Mm}{2R} - \frac{GMm}{10R} - \left( \frac{1}{2}mv^2 - \frac{G5Mm}{10R} - \frac{GMm}{2R} \right) = W_{eng}$$

$$W_{eng} = \frac{GMm}{R} \left( -\frac{5}{2} - \frac{1}{10} + \frac{5}{10} + \frac{1}{2} \right) - \frac{1}{2}mv^2$$

$$W_{eng} = -\frac{8}{5} \frac{GMm}{R} - \frac{1}{2}mv^2$$

Name: \_\_\_\_\_

8. A firecracker of mass  $3M$  travels with velocity  $V$  in the positive  $x$ -direction. It explodes and breaks into three pieces of equal masses, as shown in the diagram. Immediately after the explosion, one piece travels in the positive  $y$ -direction with speed  $\frac{1}{2}V$ , the second piece travels along the original direction at speed  $\frac{1}{2}V$ , and a third fragment travels at unknown speed in an unknown direction.



a) (40 points) Derive an expression for the **speed** of the third fragment in terms of system parameters.

$$\vec{J}_{\text{net ext}} = \vec{P}_f - \vec{P}_i$$

$$\vec{P}_i = \vec{P}_f$$

$$P_{ix} = P_{fx}$$

$$3MV = M\frac{V}{2} + MV_{3x}$$

$$V_{3x} = \frac{5}{2}V$$

$$P_{iy} = P_{fy}$$

$$0 = M\frac{V}{2} + MV_{3y}$$

$$V_{3y} = -\frac{V}{2}$$

$$V_3 = \sqrt{V_{3x}^2 + V_{3y}^2} = \sqrt{\left(\frac{5}{2}V\right)^2 + \left(\frac{V}{2}\right)^2}$$

$$V_3 = \sqrt{\frac{26}{4}}V = \frac{\sqrt{26}}{2}V$$

b) (10 points) Derive an expression for the energy released in the explosion, in terms of  $M$  and  $V$ . Simplify as far as possible.

$$\Delta E = K_f - K_i$$

$$= \frac{1}{2}M\left(\frac{V}{2}\right)^2 + \frac{1}{2}M\left(\frac{V}{2}\right)^2 + \frac{1}{2}M\left(\frac{\sqrt{26}}{2}V\right)^2 - \frac{1}{2}3MV^2$$

$$= \frac{1}{2}MV^2 \left[ \frac{1}{4} + \frac{1}{4} + \frac{13}{2} - 3 \right]$$

$$= \frac{1}{2}MV^2 [7 - 3]$$

$$\Delta E = 2MV^2$$