

Rec Sec Number \_\_\_\_\_

**TEST 3 (4 pages)**

and First Name: \_\_\_\_\_

For questions 1-5, write your answer in the underlined space **to the left of the question number**. For problems 6-8: begin with an Official Starting Equation. The final expression must be in system parameters and simplified as far as possible. Neglect air resistance. Calculators and notes cannot be used during the test. If you have questions, ask the proctor. **You must put your name on each page.**

**Test Total = 200 / 200**

A 1. (10 points) What is the moment of inertia of a uniform rod of mass  $2M$  and length  $L$  that is pivoted at a point  $\frac{1}{3}L$  from its end?

- A)  $(2/9)ML^2$       B)  $(5/9)ML^2$       C)  $(8/9)ML^2$       D)  $(4/9)ML^2$

B 2. (10 points) A large heavy disk, a hoop of same mass and radius as this disk, and a small light disk are placed at the top of an incline and released from rest. They are rolling without slipping. Which one is first to reach the bottom of the incline?

- A) large heavy disk      B) both disks at the same time  
C) small light disk      D) hoop

D 3. (10 points) A rigid object is rotating with some angular velocity  $\vec{\omega}$ . If the net torque acting on it is in the opposite direction as the angular velocity, then the angular acceleration will be

- A) be zero      B) perpendicular to  $\vec{\omega}$   
C) in the direction of  $\vec{\omega}$       D) in the direction opposite to  $\vec{\omega}$

A 4. (10 points) A cat sits at the rim of a rotating merry-go-round. A dog sits half-way between the center and the rim. Which is true about their angular speeds?

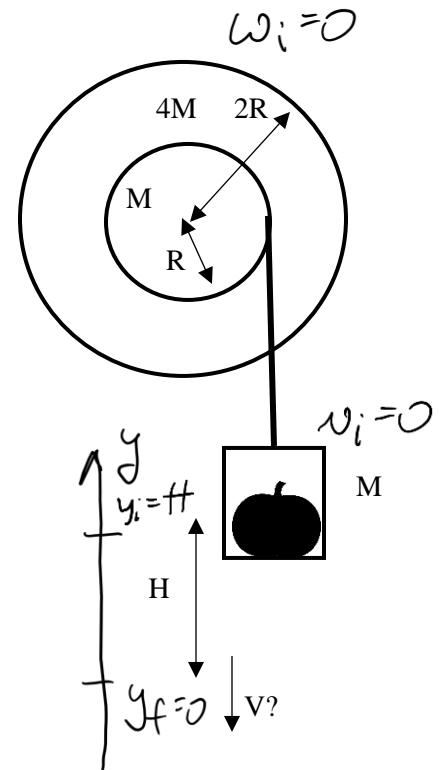
- A) The cat and the dog have the same angular speed.  
B) The cat's angular speed is larger than the dog's.  
C) The cat's angular speed is smaller than the dog's.  
D) We need to know which animal is heavier; that one will have the larger angular speed.

D 5. (10 points) In which of the following processes is the angular momentum of the system **NOT** conserved?

- A) A star collapsing into a neutron star  
B) Planetary motion under the influence of the star's gravity  
C) A figure skater stretching her arms out in a spin  
D) A uniform sphere rolling down a slope

Name: \_\_\_\_\_

6. (50 points) Two cylindrical disks, one of mass  $M$  and radius  $R$ , and one of mass  $4M$  and radius  $2R$ , are welded together and are then mounted on a **fixed** frictionless axle through their common center. A massless string is wrapped around the edge of the *smaller* disk, and a box containing a pumpkin is suspended from the free end of the string, as shown in the figure. The combined mass of box and pumpkin is  $M$ . The string does not slip on the disk. The system is released from rest.



a) (10 points) Find the combined moment of inertia of the two disks, in terms of system parameters.

$$I = \frac{1}{2}(4M)(2R)^2 + \frac{1}{2}MR^2$$

$$I = 8MR^2 + \frac{1}{2}MR^2 = \frac{17}{2}MR^2$$

b) (40 points) Use **energy methods** to derive an expression for the **speed** of the box with the pumpkin when it has descended a distance  $H$ , in terms of system parameters.

$$E_f - E_i = W_{other}$$

$$E_i = E_f$$

$$\frac{1}{2}Mv_i^2 + \frac{1}{2}I\omega_i^2 + Mgy_i = \frac{1}{2}Mv_f^2 + \frac{1}{2}I\omega_f^2 + Mgy_f$$

$$MgH = \frac{1}{2}Mv_f^2 + \frac{1}{2}I\omega_f^2$$

No slipping:  $v = \omega R$

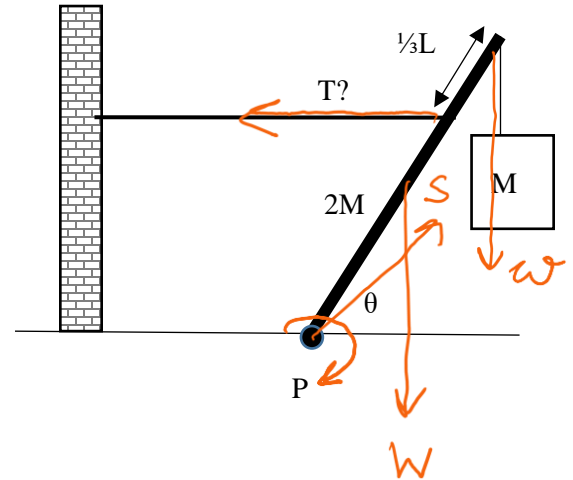
$$MgH = \frac{1}{2}Mv_f^2 + \frac{1}{2}\left(\frac{17}{2}MR^2\right)\frac{v_f^2}{R^2}$$

$$MgH = \frac{1}{2}Mv_f^2\left(1 + \frac{17}{2}\right)$$

$$v_f = \sqrt{\frac{4}{19}gH}$$

Name: \_\_\_\_\_

7. (50 points) A uniform beam of length  $L$  and mass  $2M$  is pivoted on the floor at point P. It is held in place by a horizontal cable that is anchored to a wall and attached to the beam at a distance of  $\frac{1}{3}L$  from the upper end. The beam makes angle  $\theta$  with the horizontal floor. A box of mass  $M$  is suspended from the end of the beam.



a) (10 points) Complete the diagram with all information necessary to solve parts b and c below.

b) (20 pts) Derive an expression for the tension in the horizontal cable, in terms of system parameters. (Hint: Torques about P)

$$\begin{aligned}\sum \tau_z &= \cancel{\tau_{S_z}} + \tau_{W_z} + \tau_{w_z} + \tau_{T_z} = 0 \\ 2Mg \frac{L}{2} \cos \theta + Mg L \cos \theta - T \frac{2}{3} L \sin \theta &= 0 \\ 2Mg L \cos \theta &= \frac{2}{3} T L \sin \theta\end{aligned}$$

$$T = 3Mg \cot \theta$$

c) (20 pts) Derive expressions for the horizontal and vertical components of the support force at the pivot point, in terms of system parameters. **Treat the tension  $T$  found in part b as a system parameter for this part.**

$$\begin{aligned}\sum F_x &= T_x + W_x + w_x + S_x = 0 \\ -T + S_x &= 0 & S_x &= T\end{aligned}$$

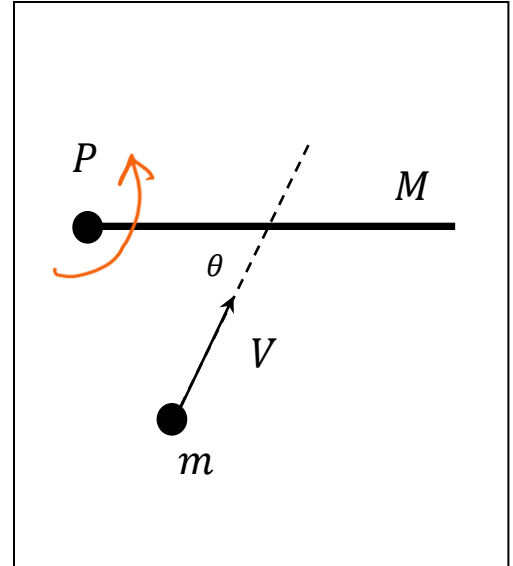
$$\begin{aligned}\sum F_y &= T_y + W_y + w_y + S_y = 0 \\ -2Mg - Mg + S_y &= 0 & S_y &= 3Mg\end{aligned}$$

Name: \_\_\_\_\_

8. (50 points) A bullet of mass  $m$  and speed  $V$  collides with a uniform rod. The rod has mass  $M$  and width  $L$  and is pivoted at point  $P$  so it can rotate freely in a horizontal plane. It is initially at rest.

The bullet strikes the rod at an angle  $\theta$  at a point located  $\frac{2}{5}L$  from the pivot. The bullet passes through the rod and continues moving in the same direction, but with reduced speed. Immediately after the collision, the rod rotates with an angular velocity  $\omega$ .

Derive an expression for the speed of the bullet immediately after the collision, in terms of system parameters.



$$\sum \vec{\tau}_{ext} = \frac{d\vec{L}}{dt}$$

$$L_{iz} = L_{fz}$$

$$mV \frac{2}{5}L \sin \theta = mV_f \frac{2}{5}L \sin \theta + \frac{1}{3}ML^2 \omega$$

$$V_f = \frac{\frac{2}{5}mV L \sin \theta - \frac{1}{3}ML^2 \omega}{\frac{2}{5}mL \sin \theta}$$

$$V_f = V - \frac{\frac{1}{3}ML^2 \omega}{\frac{2}{5}mL \sin \theta}$$

$$V_f = V - \frac{5}{6} \frac{ML\omega}{m \sin \theta}$$